Enhanced Unitary Beamforming Scheme for Limited-Feedback Multiuser MIMO Systems

Wonmu Lee, Iills Sohn, Byong Ok Lee, and Kwang Bok Lee

Abstract—In this letter, we consider practical downlink multiuser multiple-input multiple-output (MU-MIMO) systems where each user has multiple antennas and codebook-based limited channel feedback is available at base station. We propose a preprocessing scheme for downlink transmission where a unitary beamforming matrix is used. Firstly, we propose how to construct the unitary matrix that maximizes sum rate. Secondly, we propose a codebook-based channel feedback method for the proposed beamforming method. Numerical results show that the proposed scheme outperforms conventional zero-forcing beamforming scheme in terms of sum rate.

Index Terms—MIMO, downlink, broadcast channel, unitary beamforming, limited feedback.

I. INTRODUCTION

LINEAR beamforming schemes for downlink multiuser multiple-input multiple-output (MU-MIMO) systems have attracted interests because of their simplicity and near optimal sum rate performance [1]. In practical systems, since only partial channel state information (CSI) is available at a base station, most previous works have adopted codebook-based approach for quantizing CSI [2], [3].

Zero-forcing beamforming (ZFBF) is a well-known linear beamforming scheme, which only concentrates on perfectly avoiding inter-user interference at the sacrifice of signal power [1]. Although ZFBF scheme does not guarantee maximum sum rate, it shows near optimal sum rate performance when there are sufficiently large number of users and perfect CSI is available at a base station as noted in [1]. However, ZFBF scheme shows poor sum rate performance at low signal-to-noise ratio (SNR) region due to the signal power penalty. Furthermore, in practical systems, remaining inter-user interference caused by channel quantization errors significantly degrades the sum rate of ZFBF scheme. To minimize the remaining inter-user interference, the author in [2] has proposed the receive antenna combining technique, referred to as quantization-based combining (QBC), by which channel quantization error for each user is minimized. Although the sum rate of ZFBF scheme can be improved by QBC, the performance degradation due to imperfect CSI is still unavoidable.

On the other hand, a simple unitary beamforming scheme has been proposed in [3] where a base station selects the beamforming matrix that maximizes sum rate among predefined unitary matrices in a codebook. In comparison with ZFBF scheme, it has a performance limitation since it uses only small number of predefined unitary matrices. Thus, we propose a new unitary beamforming scheme to improve the conventional unitary beamforming scheme. We simplify the problem numerically. Compared with the conventional scheme, the proposed scheme flexibly constructs the unitary beamforming matrix that maximizes sum rate. Hence, since the proposed scheme has higher degree of freedom in the beamforming matrix than the conventional scheme, it is more advantageous to enhance sum rate. Simulation results are presented to compare the sum rate of the proposed scheme with that of ZFBF scheme combined with QBC with respect to the feedback rate.

The notations $\mathbf{E}[]$, $\text{tr}[]$, $\text{det}[]$, and $(\cdot)^\mathsf{H}$ stand for expectation, trace, determinant, and conjugate transpose, respectively. The symbol $\mathbf{I}_M$ denotes the $M \times M$ identity matrix.

II. SYSTEM MODEL

Fig. 1 illustrates our system model. We consider downlink MU-MIMO systems with $N_T$ antennas at a base station and $N_R$ antennas at each user. We assume that $K$ users have already been chosen in higher layers, and each selected user has single data stream, i.e., $N_T = K$. The received symbol at the $k$th user, denoted as $y_k$, can be represented as

$$y_k = r_k^\mathsf{H} (\mathbf{H}_k \mathbf{F} x + \mathbf{n}_k), \quad k = 1, 2, \cdots, N_T.$$  

(1)

$\mathbf{H}_k \in \mathbb{C}^{N_R \times N_T}$ is the independent and identically distributed (i.i.d.) flat Rayleigh fading channel matrix of the $k$th user, which is assumed to be static during a time slot and changing.
in [4], where a 2-bit codebook generated by Grassmannian line packing (GLP) is, the additive white Gaussian noise with unit variance, and \( \gamma_k \in \mathbb{C}^{N_k \times 1} \) is the combining vector which has unit norm. We assume that each user has perfect knowledge for the instantaneous channel of its own, and a feedback link is error-free and zero-delay. In this letter, we use the B-bit codebook generated by Grassmannian line packing (GLP) in [4], where 2^B quantized codeword vectors exist in the codebook C.

### III. PROPOSED UNITARY BEAMFORMING AND CHANNEL FEEDBACK METHODS

First, we propose a preprocessing method of constructing unitary beamforming matrix that maximizes sum rate when perfect CSI is available at the base station. Then, we propose a codebook-based channel feedback method for the proposed beamforming method.

#### A. Construction of Unitary Beamforming Matrix

It requires complicated studies to find the optimal linear beamforming matrix that maximizes sum rate for downlink MU-MIMO systems, since this is a non-convex problem [5]. In this section, we simplify the sum rate maximization problem using unitary beamforming condition assuming that linear minimum mean-square error (LMMSE) combining is applied at each user. As presented in [6], the post-detection SINR based on LMMSE combining of the kth user can be represented as

\[
\text{SINR}_k = \frac{1}{\left( \frac{\text{SNR}}{N_T} F_k^H H_k F_k + I_{N_T} \right)^{-1}} - 1. \tag{2}
\]

If the beamforming matrix \( F \) is unitary, from (2), SINR \( k \) can be simplified as follows using the property that \( F^H F = F F^H = I_{N_T} \):

\[
\text{SINR}_k = \frac{1}{F_k^H \left( \frac{\text{SNR}}{N_T} H_k^H H_k + I_{N_T} \right) F_k} - 1 \quad \tag{3}
\]

where \( f_k \) denotes the beamforming vector of the kth user which is the kth column vector of the beamforming matrix \( F \). From (3), we can see that SINR \( k \) only depends on the beamforming vector \( f_k \) of the kth user independent of the beamforming vectors of the other users if only all beamforming vectors are orthogonal each other. Hence, sum rate \( R \) is simply represented as

\[
R = \sum_{k=1}^{K} \log_2(1 + \text{SINR}_k) = -\sum_{k=1}^{K} \log_2 f_k^H \left( \frac{\text{SNR}}{N_T} H_k^H H_k + I_{N_T} \right)^{-1} f_k. \tag{4}
\]

#### B. Channel Feedback Method

In the previous section, the proposed beamforming method uses perfect CSI to construct a beamforming matrix. In practical systems, however, only partial CSI is available at the base station. As we can see in Table I, the base station needs the information of \( H_k^H H_k \) to construct a beamforming matrix. We represent \( H_k^H H_k = V \Sigma V^H \) by singular value decomposition \( V = U \Sigma V^H \). As a simple and effective approach, we approximate \( H_k^H H_k \) value using the vector corresponding to the maximum singular value as follows:

\[
H_k^H H_k \approx \sigma_1^2 v_1 v_1^H \quad (\text{here}, \sigma_1 > \cdots > \sigma_{N_T}). \tag{6}
\]

Among \( 2^B \) codeword vectors in the predefined codebook \( C \), the kth user selects the codeword vector, denoted as \( c_k \), which

| TABLE I
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<tr>
<th>CONSTRUCTION ALGORITHM OF UNITARY BEAMFORMING MATRIX</th>
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<tr>
<td>Step 1: Choose any ( F \in \mathbb{C}^{N_T \times N_T} ) such that ( F F^H = F^H F = I_{N_T} ). Set step size ( \gamma = 1 ). Here, ( F = [f_1, \ldots, f_k, \ldots, f_{N_T}] ).</td>
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<td>Step 2: Compute the derivative of objective function ( D_F ), where we denote ( D_F ) as the kth column vector of ( D_F ). ( D_{F,k} = \frac{\partial}{\partial F_k} \Delta_k ), where ( \Delta_k = \left( \frac{\text{SNR}}{N_T} H_k^H H_k + I_{N_T} \right)^{-1} ).</td>
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<td>Step 3: Compute the descent direction ( Z = F D_F^H F - D_F ).</td>
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<td>Step 4: Evaluate &lt; ( Z, Z \rangle = \text{tr}(Z^H (1 - \frac{1}{\text{SNR}} F F^H) Z) ). If ( \sqrt{Z} Z \frac{\gamma}{\rho} &lt; 1 ), then stop ( \rho ) (convergence bound).</td>
</tr>
<tr>
<td>Step 5: If ( L(F) - L(\pi(F + \gamma Z)) \geq \gamma &lt; Z, Z &gt; ), then set ( \gamma \leftarrow 2 \gamma ) and repeat Step 5.</td>
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<tr>
<td>Step 6: If ( L(F) - L(\pi(F + \gamma Z)) &lt; \frac{\gamma}{2} \gamma &lt; Z, Z &gt; ), then set ( \gamma \leftarrow \frac{1}{2} \gamma ) and repeat Step 6.</td>
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<td>Step 7: Set ( F = \pi(F + \gamma Z) ). Go to Step 2.</td>
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From (4), sum rate maximization problem is formulated as

\[
\text{minimize} \sum_{k=1}^{K} \log_2 \Delta_k \left( \frac{\text{SNR}}{N_T} H_k^H H_k + I_{N_T} \right)^{-1} f_k \\
\text{subject to} \quad F F^H = F^H F = I_{N_T}. \tag{5}
\]

We solve the problem numerically with the algorithm presented in [7], and it is summarized in Table I. Since the optimization in (5) is a non-convex problem, the algorithm does not guarantee global optimality. However, we show that the algorithm achieves high sum rate through simulation results.
has the minimum Euclidean distance from $v_1$:

$$c_k = \arg \max_{x \in \mathbb{C}} |v_1^H x|^2,$$

(7)

and sends its index to the base station. Then, the base station constructs a unitary beamforming matrix using the information $c_k v_k^H$ instead of $H_k^H H_k$ in the construction algorithm.

IV. NUMERICAL RESULTS

In this section, the sum rate performances for the proposed scheme and ZFBF scheme combined with QBC (ZFBF-QBC) are evaluated and compared each other. Sum rate performances are shown for a $N_T = K = 4$ and $N_R = 2$ MU-MIMO system with 4-bit, 8-bit, and 12-bit codebooks. Sum rate is averaged for 10,000 independent realizations of the channel matrices. We set the convergence bound $\rho$ to 0.001, and also apply LMMSE combining to ZFBF-QBC for fair comparison. As the upper bound of sum rate, the channel capacity through dirty paper coding (DPC) is presented using the algorithm in [8]. In the proposed scheme, sum rate performances are shown for both perfect CSI and partial CSI.

It is shown in Fig. 2 that the proposed scheme outperforms ZFBF-QBC over 5dB gain for all codebooks except for high SNR region. Since ZFBF scheme only focuses on avoiding inter-user interference at the sacrifice of signal power, ZFBF scheme shows poor sum rate performance at low SNR region. On the other hand, the proposed scheme is designed to maximize sum rate. Hence, the sum rate of the proposed scheme is higher than that of the ZFBF scheme especially at low SNR region. On the other hand, unitary beamforming schemes including the proposed scheme always cause inter-user interference due to the unitary structure of a beamforming matrix. Thus, at high SNR region, ZFBF scheme achieves higher sum rate than the proposed scheme. In practical systems, however, since the system operating point of SNR is lower than 20dB in most cases, the proposed scheme is generally more advantageous than ZFBF-QBC scheme. Moreover, since the inter-user interference can be effectively suppressed by an advanced receive antenna combining technique such as LMMSE combining, the disadvantage of the unitary structure vanishes as more antennas are available at each user.

V. CONCLUSIONS

We propose a new preprocessing method utilizing unitary beamforming matrix for downlink MU-MIMO systems, and codebook-based channel feedback method for the proposed beamforming method. The proposed scheme flexibly constructs the unitary beamforming matrix that maximizes sum rate, while ZFBF scheme only focuses on avoiding inter-user interference. Simulation results have verified that the proposed scheme provides significant sum rate gain over ZFBF scheme in limited feedback environments.

REFERENCES


Fig. 2. Sum rate versus SNR, where $N_T = 4$, $N_R = 2$, and $K = 4$. 

平均幂率 [bps/Hz]