Blind Beamforming Robust to Directional Error in Fixed Wireless Channel

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ABSTRACT

Some blind beamformers require the direction of the desired signal. The performance of such type beamformers can be severely degraded by a small directional error. In this paper, we propose a blind beamforming scheme robust to directional error by employing a simple steering vector estimator. The performance of the proposed beamformer can be further improved by re-initializing the weights of the beamformer.

I. INTRODUCTION

In recent years, exploding demand for wireless communications has led to the use of advanced transceiver techniques to increase the transmission capacity without increasing the bandwidth. One of the advanced techniques is adaptive beamforming, which was proposed to reduce the multipath fading effect and co-channel interference [1-4]. The signal to interference plus noise ratio (SINR) of the received signal can be improved by the use of beamforming, increasing the system capacity [1-2]. The most advanced beamforming technique for increasing the capacity is spatial division multiple access (SDMA) technique [3-4]. The SDMA concept can be easily applied to fixed wireless systems because the direction of subscribers is fixed.

In fixed wireless systems, we can use a linear constraint minimum variance (LCMV) beamformer without the need of a complicated direction-of-arrival (DOA) estimator such as the multiple signal classification (MUSIC) method because the location of subscribers is available. However, there can be small error in directional information and this error will degrade the performance of the beamformer. Recently, a simple and efficient blind beamforming scheme was proposed by employing a spatial prefilter assuming that the direction of the desired signal is known [5]. This prefilter-based scheme is a kind of minimum variance (MV) beamformers whose prefilter coefficients are determined by the direction of the desired signal. However, the SINR performance of this beamformer can be severely degraded in an interference dominant environment if there exists directional error.

In this paper, we consider the improvement of the prefilter beamformer by making it robust to directional error. We can use an adaptive signal canceller as a simple steering vector estimator, instead of using a complicated DOA estimator. If the signal canceller is applied to subtracting the output of the beamformer from its input signal, the coefficients of the signal canceller are converged to the steering vector of the desired signal [6][7]. Using these coefficients, the error of the spatial prefilter can be corrected. We also consider further performance improvement by re-initializing the weights of the beamformer. The decision procedure for need of the re-initialization is analytically designed using the error signal in the estimator.

II. SIGNAL MODEL

We can consider the use of an N-element uniformly spaced linear array (ULA) for the purpose of beamforming. Assume that there are L users in communications and that the signal of each user can be modeled as a planar wave arriving to the ULA. The received baseband equivalent signal at the n-th element of the ULA can be represented by

\[ x_n(t) = \sum_{l=1}^{L} a_n(\theta_l) \beta_l(t) r_l(t - r_l) + n_n(t), \quad 1 \leq n \leq N \] (1)

where \( a_n(\cdot) \) is the n-th component of the steering vector \( \mathbf{a} \) given by

\[ \mathbf{a}(\theta_l) = [1, e^{-j2\pi(d/\lambda)\sin(\theta_l)}, \ldots, e^{-j(N-1)2\pi(d/\lambda)\sin(\theta_l)}]^T. \] (2)

Here \( d \) is the distance between the antennas, \( \lambda \) is the wavelength, the superscript \( T \) denotes the transpose of a vector and \( \theta_l \) is the direction of the l-th user. \( \beta_l(t) \) is the complex envelope fading of the l-th user signal and \( r_l(t) \) is represented as...
where \( b_{ij} \) is the data of the \( l \)-th user, \( g(t) \) is the impulse response of the pulse shaping, and \( T_r \) is the symbol period. The time delay \( \tau_i \) is the delay of \( l \)-th user signal and \( n_i(t) \) is the additive noise. In a vector form, the received signal from the antenna array can be represented by

\[
x(t) = \sum_{l=1}^{L} a(\theta_l) \beta_l(t) r_l(t - \tau_l) + n(t)
\]

(4)

where \( n(t) = \left[ n_1(t), n_2(t), \ldots, n_L(t) \right] \).

Assuming that there exists a line-of-sight (LOS) during the transmission, the channel can be modeled as a Rician fading channel. Then the received signal sampled at time \( t = kT_r \) can be represented by

\[
x[k] = \sum_{l=1}^{L} \beta_l[k] s_l[k a(\theta_l)] + n[k]
\]

(5)

where \( s_l[k] \) is assumed to be the desired signal, \( s_l[k] \)'s, \( l = 2, 3, \ldots, L \), are the interferences.

### III. THE PREFILTER BEAMFORMER

A block diagram of the proposed beamformer is depicted in Fig.1, where the prefilter beamformer in [5] is shown in the dotted rectangular box. The prefilter with coefficient \( C_a \) behaves as a beamformer whose output \( y[k] \) has a zero gain in the direction of the desired signal. Therefore the prefilter beamformer \( C_a \) can be considered as a data independent one. The \((m, n)\)-th coefficient of the prefilter \( C_a \) is given by [5]

\[
(C_a)_{mn} = \begin{cases} 
1, & m = n \\
-p_1, & m = n+1 \\
0, & \text{otherwise}
\end{cases}
\]

(6)

where \( p_1 = e^{-j2\pi d / \lambda \sin \theta} \), \( m = 1, 2, \ldots, N \) and \( n = 1, 2, \ldots, N \). The output of the prefilter is given by

\[
v[k] = C_a^H x[k]
\]

(7)

\( i.e. \), each component of \( v[k] \) is obtained by

\[
v_n[k] = x_n[k] - p_1^* x_{n+1}[k], \quad n = 1, \ldots, N - 1
\]

(8)

where the superscripts * and \( H \) denote the complex conjugate and the Hermitian of a matrix, respectively.

Let \( w[k] \) be the weighting vector of the beamformer. Note that both \( v[k] \) and \( w[k] \) have the same dimension equal to \((N - 1)\). The spatial gain in direction \( \theta \) is

\[
G_{\theta}(\theta, k) = w[k]^H C_a a(\theta)
\]

\[
= \left(1 - p_1^* e^{-j2\pi d / \lambda \sin \theta} \right) \sum_{n=1}^{N-1} w_n[k] e^{-j(n-1)2\pi d / \lambda \sin \theta}
\]

(9)

It can be seen that any signal from direction \( \theta \) is filtered out by the prefilter. Since \( z[k] \) does not contain any desired signal component, the optimum least mean square (LMS) weight \( w[k] \) can be obtained by minimizing \( |z[k]|^2 \), \( i.e., \)

\[
w[k] = w[k] - \mu \cdot z[k]^H C_a^H x[k]
\]

(10)

where \( \mu \) is the step size for adaptation.

Since the dimension of \( w[k] \) is less than that of \( x[k] \) by one, the input \( x[k] \) needs to be preprocessed by \( C_a \) whose \((m, n)\)-th element is given by

\[
(C_a)_{mn} = \begin{cases} 
1, & m = n \\
0, & \text{otherwise}
\end{cases}
\]

(11)

where \( m = 1, 2, \ldots, N \) and \( n = 1, 2, \ldots, N \). The adapted weight vector \( w[k] \) is applied to the original received signal vector \( x[k] \), yielding the output

\[
y[k] = w[k]^H C_a x[k].
\]

(12)

The spatial gain of the output \( y[k] \) in direction \( \theta \) is

\[
G_{\theta}(\theta, k) = w[k]^H C_a a(\theta)
\]

\[
= \sum_{n=1}^{N-1} w_n[k] \exp^{-j(n-1)2\pi d / \lambda \sin \theta}
\]

(13)

### IV. BEAMFORMING ROBUST TO DIRECTIONAL ERROR

The directional information of each subscriber is assumed available at the base station in a fixed wireless system. If the directional information is accurate, the prefilter beamforming scheme can be easily employed without the need of a DOA estimator of the signal. However, if there is directional error, the output SINR performance can be severely degraded. Thus, it may be desirable to use a beamformer robust to directional error, without the need of accurate DOA estimation.

When there exists a directional error \( \Delta \theta \), the coefficient \( p_1 \) of the prefilter becomes
\[ p_i = \exp[-j2\pi(d/l)\sin(\theta_i + \Delta \theta)]. \]  

(14)

When the beamformer is adjusted to minimize the power of \( z[k] \), the directional error \( \Delta \theta \) results in degradation of the output SINR of the beamformer. This problem can be alleviated by correcting the coefficient \( p_i \).

To correct \( p_i \) without the need of accurate direction estimation, we consider the use of an adaptive signal canceller [6], depicted in the solid-line box in Fig.1. Define by the error vector

\[ \epsilon[k] = x[k] - h[k]y[k] \]  

(15)

where \( h[k] \) represents \( N \) coefficients of the canceller. The coefficient \( h[k] \) can be updated by minimizing the power of error vector \( \epsilon[k] \),

\[ h_n[k+1] = h_n[k] - \frac{\mu_e}{2} \frac{\partial E\{\epsilon^2[k]\}}{\partial h_n} \]  

(16)

where \( n=1, 2, \cdots, N \). \( \mu_e \) is the step size for adaptation and \( E\{u\} \) denotes the ensemble average of \( u \).

Assuming that the output \( y[k] \) is nearly equal to the desired signal \( s[k] \) in the steady state, and that there is no correlation between the signal and the interferences, it can be shown that the optimum MMSE coefficients \( \hat{h} \) are given by

\[ \hat{h}_n = \frac{\sigma^2_n}{\sigma^2_s} \exp\left[-j(n-1)2\pi\left(\frac{d}{\lambda}\right)\sin\theta_\theta\right] \]  

(17)

where \( n=1, 2, \cdots, N \). \( \sigma^2_n \) is the power of \( s[k] \) and \( \sigma^2_s \) is the variance of \( y[k] \). Since the ensemble average is not available in real time, the coefficients can be updated by

\[ h[k+1] = h[k] + \mu_e \epsilon[k]y[k] \]  

(18)

When the coefficients \( h[k] \) of the canceller converge to \( \hat{h} \), it can be seen that

\[ \lim_{h \to \hat{h}} \frac{h_n[k]}{h_n^{-1}[k]} = \hat{h}_n \]  

\[ = e^{j\left(-2\pi(d/l)\sin\theta_\theta\right)} \]  

(19)

where \( n=2, \cdots, N \). Since there are \( (N-1) \) ratio terms, the prefilter coefficient can be estimated by averaging the ratio terms.

\[ \hat{p}_t[k] = \frac{1}{N-1} \sum_{n=1}^{N-1} \frac{h_n[k]}{h_n^{-1}[k]} \]  

(20)

Note that the magnitude of \( \hat{p}_t[k] \) is normalized to the unity. Thus, the directional error can be corrected without the use of a complicated direction estimator.

Re-initialization of the weights of the beamformer: Since the weight vector \( w[k] \) of the prefilter beamformer is optimized for rejection of interferences, it is obtained without any constraint on the desired signal. To make the beamformer rapidly converge to the steady state, the weight vector is initialized to a value corresponding to the steering vector \( a(\theta_\theta) \). When there exists directional error \( \Delta \theta \), the weight vector will be initialized with respect to \( a(\theta_\theta + \Delta \theta) \). The proposed method adjusts the weight vector to make nulls in the directions of interferences, but it cannot make the desired signal have a maximum gain. This problem can be alleviated by re-initializing the weight vector \( w[k] \).

Since the coefficient \( h \) is proportional to \( a(\theta_\theta) \), the weight vector can be re-initialized by \( w[k] = h[k] / |h[k]| \).

Since the mean squared error (MSE) of the signal canceller decreases as \( h[k] \) converges to \( \hat{h} \), it can be used as a test statistic to decide whether the weight vector needs to be re-initialized or not. Note that the MSE value of \( \epsilon[k] \) converges very slowly because \( \epsilon[k] \) contains interference signals.

The test statistic \( T[k] \) can be obtained by averaging the error signals \( \epsilon_n[k], n=1, \cdots, N \), to reduce the interference effect

\[ T[k] = E\left\{\frac{1}{N} \sum_{n=1}^{N} \epsilon_n^2[k]\right\}. \]  

(21)

Fig.2 depicts the transient response of the test statistic \( T[k] \).

To make the beamformer optimum, it is desirable to re-initialize the weight vector after \( h[k] \) converges to \( \hat{h} \), but it may take long time. Since the performance degradation is not so severe for directional error less than one degree, it would be practical to re-initialize the weight vector after \( T[k] \) is smaller than a threshold value. Let \( \theta_m \) be a directional error that results in acceptable performance degradation. Then the threshold \( \eta \) can be determined by a value of \( T[k] \) when \( \Delta \theta = \pm \theta_m \).

When the directional error is equal to \( \theta_m \), the output signal can be modeled by \( y[k] = \alpha s[k] + q[k] \), where \( \alpha \) is the spatial gain in the direction of \( (\theta_\theta \pm \theta_m) \) and \( q[k] \) is the noise term at the output of the beamformer. Assuming that the signal, noise and interferences have zero mean and that they are uncorrelated with one another, the threshold \( \eta \) can be determined by
$$\eta = E\{T[k]\}_{\theta_{0}=\pm \theta_{0}}$$
$$= \sigma_{q}^{2} \left[ \frac{1}{N} \sum_{n=1}^{N} \left( e^{j(n-1)2\pi d/L \sin(\theta_{0})} - \alpha e^{j(n-1)2\pi d/L \sin(\theta_{0})} \right)^{2} \right]$$
$$+ \sigma_{q}^{2} \left[ \frac{1}{N} \sum_{n=1}^{N} e^{-j(n-1)2\pi d/L \sin(\theta_{0})} \right]^{2}$$

$$+ \sigma_{i}^{2}$$  \hspace{1cm} (22)

where \( T[k] \) is the term due to noise \( q[k] \) and interferences, \( \sigma_{q}^{2} \) is the variance of \( T[k] \), \( \sigma_{i}^{2} \) is the variance of \( q[k] \). The threshold level \( \eta \) is determined by the smaller one between the two values in (22).

To evaluate the performance of the proposed beamformer, we consider a fixed wireless transmission system operating at 2.4 GHz band. The gain in the fixed wireless channel varies very slowly with Doppler frequency less than a few Hz, yielding the path fading \( \beta(f) \) nearly constant during the adaptation. Assume that the desired signal is received from direction 0° and four interferences are received respectively from 60°, -40°, 30° and 45°, where the desired signal and interferences are all QPSK signals having the same power at a symbol rate of 160 Kbaud. The power of the additive noise is 20dB below the signals.

Fig.3 depicts the SINR performance of the beamformer with the use of an 8-element ULA with half wavelength spacing. It can be seen that the proposed scheme can provide the SINR performance quite robust to a wide range of directional error by updating the coefficient of the prefilter. To determine the need of re-initialization of the weight vector, \( T[k] \) is calculated using a 20-tap moving average filter as shown Fig. 4. The threshold \( \eta \) is determined assuming that \( \sigma_{i} = 0 \) which results in the smallest threshold value for conservative design. Since it takes additional time to re-initialize the weight as seen in Fig. 5, it may be useful to use the re-initializing method only when the directional error becomes significant.

V. CONCLUSIONS

When the DOA of the desired signal is known, the use of an MV type prefilter-based blind beamformer can provide good performance. However, the SINR performance of the prefilter beamformer can be degraded by a small directional error. We have proposed a prefilter-based blind beamformer robust to directional error. The coefficient of the prefilter is corrected without the use of a complicated direction estimator. The weight of the beamformer can be re-initialized to obtain further performance improvement at the expense of a small additional updating time. Numerical results show that the proposed beamformer can provide SINR performance quite robust to a wide range of directional error, indicating that it can be applied to wireless communications where the desired signal is slowly moving, e.g., pedestrians.

REFERENCES


Fig. 1 A block diagram of the proposed beamformer
Fig. 2 The transient response of the test statistic $T[k]$

Fig. 3 SINR performance when directional error exists.

Fig. 4 The decision procedure for re-initialization

Fig. 5 Convergence of the proposed beamformer when $\Delta \theta = 3^\circ$