I. INTRODUCTION

Frequency-hopping spread-spectrum (FHSS) systems have been widely used in military communications. Demands for high data rate services in FHSS systems have been increasing. In high data rate systems, the effects of frequency-selective fading should be considered due to an increase in the ratio of delay spread to symbol duration. The effects of frequency-selective fading on an FHSS system employing orthogonal binary frequency-shift keying (BFSK) signal are investigated in [1] and [2] under the assumption that the frequency separation between two orthogonal BFSK signals is much larger than the minimum required to guarantee orthogonality, such that the correlation between two correlator outputs in neglected. In practice, it is advantageous to use the minimum frequency separation in multiple access environments to increase the number of frequency slots for given total bandwidth [3]. When the minimum frequency separation is employed, correlation between two correlator outputs due to frequency-selective fading and fast fading may be significant. This paper does not assume that the correlation is neglected.

Transmission diversity provides protection against jamming, multiple access interference and fading. For FHSS systems, the diversity may be realized in the form of fast frequency-hopping (FFH) and multicarrier transmission. Fast frequency-hopping (FFH) is a conventional diversity technique in FHSS systems; multicarrier transmission is an alternative diversity technique in FHSS systems. FFH systems have attracted considerable interest and their performance has been widely studied over the past few decades. Recently, a multicarrier transmission technique has been proposed and the use of MCFH has been investigated for coherent FHSS systems employing binary phase-shift keying (BPSK) in [4]. However, in FHSS systems, coherent demodulation for BPSK signal is relatively difficult. FSK modulation with noncoherent detection is typically employed in FHSS systems [1][3][5]. Hence, in this paper, multicarrier frequency-hopping (MCFH) system with BFSK modulation and noncoherent demodulation is investigated. In this system, the total frequency band is partitioned into several disjoint subbands on which replicas of the same signal are simultaneously transmitted. Each replica hops independently in its subband. Block diagrams of the FFH and that of the MCFH systems are respectively shown in Figs. 1 and 2. For systems employing diversity, diversity receptions should be combined in some way in the receiver. A number of diversity combining schemes for FFH systems have been developed and their performances have been studied [5][6]. These combining schemes can also be applied to MCFH system. The optimum diversity combining scheme is investigated in this paper. The optimum combining rule is obtained by the use of the likelihood functions. For slow and frequency-nonselective Rayleigh fading channels, the optimum combining rule is investigated in [6]. In this paper, the optimum combining rule for frequency-selective fast varying Rayleigh fading channels is developed.

Based on the developed optimum diversity combining rule, bit error rate (BER) equations are derived. The BER performance of these two systems is evaluated and compared for various channel conditions, and the effects of diversity order on performance are investigated. Furthermore, the effects of frequency-selective fading on the optimum value of the frequency deviation are investigated. This paper is organized as follows. Section II describes the system and channel models. In Section III, optimum diversity combining rule and equations for the probability of error are derived. In Section IV, performance evaluation is presented and performance comparisons between FFH and MCFH systems are made. Finally, conclusions are summarized in Section V.

II. SYSTEM AND CHANNEL MODELS

The systems considered in this paper are FHSS systems with BFSK modulation, noncoherent detection and diversity order $L$. The diversity order means the number of
hops per symbol for FFH system and the number of subbands for MCFH system. It is assumed that each diversity reception is independently Rayleigh faded. The maximum delay spread of each diversity reception is assumed to be smaller than the hop duration of FFH systems, which is smaller than the symbol duration. We also assume that one symbol is transmitted during one hop duration in MCFH systems, and adjacent symbols in time are transmitted in far distant frequency slots such that multipath interference from the previous symbol is negligible.

Transmitter block diagrams of FFH and MCFH systems are, respectively, depicted in Figs. 1(a) and 2(a). The binary data is FSK modulated and hopped to \( L \) different hop frequencies generated by a frequency synthesizer or a bank of frequency synthesizers, which are controlled by pseudonoise codes. The complex baseband equivalent of the transmitted signal for each system can be represented as

\[
s(t) = \sum_{l=0}^{\infty} \sum_{k=-1}^{1} e^{j2\pi f_{dk} t} p_{S}(t - kT_{S}); \quad FFH
\]

\[
= \sum_{l=0}^{\infty} \sum_{k=-1}^{1} e^{j2\pi f_{dk} t} p_{S}(t - kT_{S}); \quad MCFH
\]

where \( S \) is the transmit power of each diversity transmission, \( T \) is symbol duration, and \( T_{S} \) is hop duration. \( f_{dk} \) and \( \phi_{dk} \) are respectively hop frequency and random phase for the \( l \)th diversity transmission of the \( k \)th symbol. \( b_{k} \in \{-1, +1\} \) is the \( k \)th data symbol, and \( p_{S}(t) = 1 \) for \( t \in [0, T] \) and zero, otherwise. Frequency deviation of BFSK signal is denoted by \( f_{d} = h / 2T_{S} = \Delta f / 2 \), where \( h \) is the normalized frequency deviation and \( \Delta f \) is the frequency separation between two BFSK signals. Note that we have used the same symbols for FFH and MCFH systems in (1). When \( s(t) \) has total transmit power \( S \), the value of \( S \) for an FFH system is \( S \), and that of \( S \) for an MCFH system is \( S/\ell \). Similarly, the value of \( T_{S} \) for the FFH system is \( T/\ell \) and that of \( T_{S} \) for the MCFH system is \( T \). Correspondingly, the values of \( f_{d} \) and \( \Delta f \) would be different for the two systems.

The channel model is a wide sense stationary uncorrelated scattering (WSSUS) model, described in [1] and [7]. The lowpass equivalent impulse response of the \( l \)th diversity channel may be written as

\[
c_{l}(t; \tau) = \alpha_{l}(t; \tau) e^{j\phi_{l}(t; \tau)}, \quad l = 0, 1, ..., L-1
\]

where \( \alpha_{l}(t; \tau) \)'s are independent and identically distributed (i.i.d.) Rayleigh random processes and \( \phi_{l}(t; \tau) \)'s are i.i.d. uniform random processes over \( [0, 2\pi] \). The autocorrelation function of the WSSUS channel is given as

\[
R_{l}(\Delta t; \tau, \tau') = \frac{1}{2} \mathbb{E}[\alpha_{l}(t; \tau)e^{j(\tau + \Delta t; \tau')}] = R_{l}(\Delta t; \tau)\delta(\tau - \tau')
\]

where \( \delta(\cdot) \) denotes a complex conjugate operation. Since channel response for each diversity transmission is assumed to be i.i.d., the autocorrelation of the channel is the same for all \( l \), so that the subscript \( l \) is dropped in (3). If we let \( \Delta t = 0 \) in \( R_{l}(\Delta t; \tau) \), the resulting autocorrelation function, denoted by \( \Phi_{l} (\tau) \), is a multipath intensity profile. Assuming that the multipath intensity profile is time-invariant, \( R_{l}(\Delta t; \tau) \) may be expressed as

\[
R_{l}(\Delta t; \tau) = \Phi_{l} (\tau)\Phi(\Delta t)
\]

where \( \Phi(\Delta t) \) is the autocorrelation function in the \( \Delta t \) variable normalized by \( \Phi_{l} (\tau) \) for all \( \tau \).

III. PERFORMANCE ANALYSIS

A. Correlator Outputs and their Statistics

Receiver block diagrams are shown in Figs. 1(b) and 2(b). After down-converting and dehopping, the complex baseband equivalent of the received signal over the first symbol duration may be expressed as

\[
r(t) = \sum_{l=0}^{\ell} \sum_{k=-1}^{1} \sqrt{2S_{l}} \alpha_{l}(t; \tau) e^{j(\phi_{l}(t; \tau) + \phi_{dk}(t))} dt + n_{l}(t); \quad FFH
\]

\[
r(t) = \sum_{l=0}^{\ell} \sum_{k=-1}^{1} \sqrt{2S_{l}} \alpha_{l}(t; \tau) e^{j(\phi_{l}(t; \tau) + \phi_{dk}(t))} dt + n_{l}(t); \quad MCFH
\]
where \( t \in [0, T] \). \( \theta(t; \tau) = \zeta(t; \tau) + \phi_{\omega_0} \) and \( n(t) \) is a lowpass equivalent AWGN process with power spectral density \( N_0 \). We assume that the data symbols \( b_i \) is either +1 or −1 with equal probability. Without loss of generality, it is assumed that data symbol \( b_0 \) is +1 hereafter. Each diversity signal is demodulated by a square-law envelope detector, which is shown in Fig. 3. We assume that the receiver is time synchronous to the signal with zero excess time (\( \tau = 0 \)). The two correlator outputs of the \( l \)th diversity reception are respectively denoted by \( R_{l, 1} \) and \( R_{l, -1} \) and may be expressed as

\[
R_{l, 1} = \frac{1}{T} \int_0^T \int_0^\infty \int_0^\infty \sqrt{2S} \alpha_l(t; \tau) e^{j\beta_0(t; \tau)} dt d\tau + \frac{1}{T} \int_0^\infty n(t) e^{j\omega_0 t} dt
\]

(6)

\[
R_{l, -1} = \frac{1}{T} \int_0^T \int_0^\infty \int_0^\infty \sqrt{2S} \alpha_l(t; \tau) e^{j\beta_0(t; \tau)} e^{j\pi \omega_0 t} dt d\tau + \frac{1}{T} \int_0^\infty n(t) e^{j(\omega_0 t + \pi)} dt.
\]

(7)

In static environments, \( R_{l, 1} \) is zero if an orthogonal BFSK is employed. However, in fading environments, \( R_{l, 1} \) is not zero, since multipath signal components and signal variation over one hop duration may destruct orthogonality. This effect is represented as the first term of (7), which will be referred to as an interference component in this paper. The second term in (6) and (7) represents AWGN. Since all terms in (6) and (7) are zero-mean complex Gaussian random variables, \( R_{l, 1} \) and \( R_{l, -1} \) are also zero-mean complex Gaussian random variables whose variances and correlation coefficient are given by

\[
\sigma^2_{1,1} = \frac{1}{2} \mathbb{E}[|R_{l, 1}|^2] = \frac{2S}{T} \int_0^\pi \int_0^{2\pi} R_l(t; \tau) \left(1 - \frac{t + \tau}{T}\right) d\tau + \frac{N_0}{T}
\]

(8)

\[
\sigma^2_{1, -1} = \frac{1}{2} \mathbb{E}[|R_{l, 1}|^2] = \frac{2S}{T} \int_0^\pi \int_0^{2\pi} R_l(t; \tau) \cos(2\pi \tau) \left(1 - \frac{t + \tau}{T}\right) d\tau + \frac{N_0}{T}
\]

(9)

\[
\rho = \frac{1}{2} \mathbb{E}[R_{l, 1} \overline{R}_{l, -1}] / [\sigma_{1,1} / \sigma_{1, -1}]
\]

(10)

As shown in Fig. 1(b), 2(b), and 3, decisions are made based on \( L \) pairs of square-law envelope detector outputs \( Z_{l, 1} \) and \( Z_{l, -1} \) for \( l \in \{0, 1, \ldots, L-1\} \). They should be combined in some way to make a decision in the receiver.

\section*{B. Optimum Diversity Combining Rule}

To find the optimum diversity combining rule based on the maximum likelihood criterion, we should find the conditional joint probability density function (PDF) of the square-law envelope detector outputs conditioned on the transmitted data symbol. The PDF is referred to as a likelihood function. Since each diversity reception is assumed to be independent of each other, the likelihood function for data symbol \( b_l = +1 \) can be expressed as

\[
Pr(z_{l, 1}, \ldots, z_{L, 1}, z_{l, -1}, \ldots, z_{L, -1} | b_l = +1)
\]

(11)

where \( Pr(z_{l, 1}, \ldots, z_{L, 1} | b_l = +1) \) is the conditional joint PDF of square-law envelope detector outputs for the \( l \)th diversity reception. This joint PDF can be easily found using the joint PDF of the correlated Rayleigh random variables given in [6], if the variances of \( Z_{l, 1} \) and \( Z_{l, -1} \) are the same. However, the variances of \( Z_{l, 1} \) and \( Z_{l, -1} \) are different in our problem as shown in (8) and (9). Hence, the results in [6] cannot be applied. To find the joint PDF of \( Z_{l, 1} \) and \( Z_{l, -1} \), the complex Gaussian random variables \( R_{l, 1} \) and \( R_{l, -1} \) are expressed in terms of inphase and quadrature components:

\[
R_{l, 1} = X_{l, 1} + jY_{l, 1}, \quad R_{l, -1} = X_{l, 1} - jY_{l, 1}
\]

(12)

where \( X_{l, 1}, X_{l, -1}, Y_{l, 1}, \) and \( Y_{l, -1} \) are zero-mean jointly Gaussian random variables with the covariance matrix

\[
E[UU^H] = \begin{bmatrix}
\sigma^2_c & \rho \sigma_c \sigma_i & 0 & \rho \sigma_c \\
\rho \sigma_i \sigma_c & \sigma_i^2 & -\rho \sigma_i \sigma_c & 0 \\
0 & -\rho \sigma_i \sigma_c & \sigma_c^2 & 0 \\
0 & 0 & 0 & \sigma_i^2
\end{bmatrix}
\]

(13)

where \( U_i \) is a column vector defined by \( U_i = [X_{i, 1}, X_{i, -1}, Y_{i, 1}, Y_{i, -1}] \), and \( \rho_c \) and \( \rho_i \) are, respectively, the real and imaginary components of the complex correlation coefficient \( \rho \) defined in (10). The superscript \( t \) denotes a vector transpose operator and \( E[\cdot] \) denotes a statistical expectation. Thus, the joint PDF of \( U_i \), conditioned on \( b_l = +1 \) is given as

\[
Pr(x_{i, 1}, x_{i, -1}, y_{i, 1}, y_{i, -1} | b_l = +1) = \frac{1}{(2\pi)^2 \sigma_c \sigma_i} \exp \left[-\frac{1}{2} \left( \frac{x_{i, 1}^2 + y_{i, 1}^2}{\sigma_c^2} \right) + \frac{1}{2} \left( \frac{x_{i, -1}^2 + y_{i, -1}^2}{\sigma_i^2} \right) \right] \cdot \exp \left[-\frac{1}{2} \left( \frac{(x_{i, 1} - y_{i, 1})^2}{\sigma_c^2} \right) + \frac{1}{2} \left( \frac{(x_{i, -1} - y_{i, -1})^2}{\sigma_i^2} \right) \right] 
\]

(14)

The PDF is expressed in terms of rectangular coordinate elements. A transformation can be made from the rectangular coordinate onto the polar coordinate via the change of variables:

\[
R_j = \sqrt{x_j^2 + y_j^2}, \quad \Psi_j = \tan^{-1}(y_j / x_j), \quad j = 1, -1
\]

(15)

\( \Psi_{-1} \) and \( \Psi_{+1} \) are uniform random variables over \( [0, 2\pi) \). By averaging the conditional joint PDF of the new random variables \( \{R_j\}, \{R_{-1}\}, \{\Psi_{+1}\}, \{\Psi_{-1}\} \) over \( \Psi_{+1} \) and \( \Psi_{-1} \), we can obtain the conditional joint PDF of amplitudes \( \{R_j\} \) and \( \{R_{-1}\} \). Finally, another transformation is made via change of variables:

\[
Z_{j, 1} = |R_j|, \quad Z_{j, -1} = \frac{R_j}{|R_j|}
\]

(16)

Then, the conditional joint PDF of square-law envelope detector outputs for the \( l \)th diversity reception is calculated as

\[
Pr(z_{j, 1}, z_{j, -1} | b_l = +1) = \int_{0}^{\infty} \int_{0}^{2\pi} \left( \frac{1}{\pi \sigma_z} \right)^{2} \exp \left[-\frac{z_{j, 1}^2}{\sigma_z^2} \right] \cdot \frac{1}{2\pi} \sigma_z \exp \left[-\frac{z_{j, -1}^2}{\sigma_z^2} \right] |E_{\pi}(\cdot)|
\]

(16)

where \( E_{\pi}(\cdot) \) is the zeroth order modified Bessel function of the first kind. Similarly, the likelihood function for data symbol \( b_l = -1 \) will be obtained using (11) and (16), if we exchange \( \sigma_c \) and \( \sigma_i \) in (16).

After straightforward algebraic manipulation and extraction of common terms in the log-likelihood functions, the optimum decision rule is derived as

\[
\sum_{l=0}^{L-1} z_{l, 1} < \sum_{l=0}^{L-1} z_{l, -1}
\]

(17)
It is interesting to note that the optimum combining rule is to simply sum the detector outputs of each diversity reception, which is the same result as the case in which orthogonality between BFSK signals is maintained [6].

C. Probability of Error

The probability of error for the optimally combined signal may be expressed as

$$P_e = \Pr \left( \sum_{i=0}^{L-1} Z_{1i} < \sum_{i=0}^{L-1} Z_{0i} \right) = \left\{ \begin{array}{ll}
1 & \text{for } \gamma = 0 \text{ and } \sigma = b \\
\frac{1}{1+\gamma} & \text{for } \gamma > 0 \text{ and } \sigma = b
\end{array} \right. ,$$

This probability can be calculated using (11) and (16) with appropriate transformation of random variables, but this work is very involved and the solution is not concise. It can be shown that (18) is the probability of error equation of a BFSK signal, similar results are obtained and not concisely presented to obtain a simple closed-form expression for the probability of error. However, direct application of (B-21) in [7] is impossible since the expression for the probability of error. However, direct application of (B-21) in [7] is impossible since the expression for the probability of error, which is troublesome in the calculation of $a/b$ and $b/a$. So, we start from (B-12) in [7] with suitable substitutions of parameters. The contour integral can be calculated using Cauchy’s theorem and Residue theorem given in [8]. Thus, the probability of error expression in (18) may be simplified to

$$P_e = \sum_{k=0}^{L-1} \frac{(2L-1)}{k} \gamma^k (1+\gamma)^{-L} ,$$

where

$$\gamma = \frac{1}{2} \left( \sigma_1^2 - \sigma_0^2 + \sqrt{\sigma_1^4 + 4\sigma_0^2 \sigma_1^2} \right) - 4\rho^2 \sigma_0^2 \sigma_1^2 .$$

IV. PERFORMANCE EVALUATION

The BER performance of FFH and MCFH systems is evaluated in this section using (19) and (20). The variances and correlation coefficient in (8), (9), and (10), are required for (19), and calculated by Monte-Carlo integration technique [9]. The autocorrelation function of a fading channel in (4) is assumed to be described by an exponential multipath intensity profile and Jakes’ fading model [10]:

$$R_r(\Delta t; \tau) = \frac{\mu e^{-\mu \Delta t}(e^{-\tau} - e^{-\mu \Delta t})}{1 - (1 + \mu) e^{-\mu \Delta t}} J_0(2\pi f_d \Delta t) .$$

where $\mu$ is a decaying factor and set to 0.5 in this paper, $f_d$ is the maximum Doppler spread, and $J_0(\cdot)$ is the zeroth order Bessel function.

Figures 4 and 5 show the performance of the FFH and MCFH systems for several values of maximum delay spreads. Figures 4 and 5 are respectively associated with the normalized maximum Doppler spread $f_d T = 0.01$ and 0.1. Diversity order $L$ is set to 3. The performance of FFH systems is found to be significantly degraded in frequency-selective fading environments with delay spread. It is also shown that the performance degradation due to delay spread is much severer in FFH systems than in MCFH systems. It can be explained as follows. It can be easily proved that the probability of error in (19) is a monotonically decreasing function of $\gamma$, which can be treated as a performance measure. The effective delay spread is defined as the ratio of the maximum delay spread to the hop duration, i.e., $T_m/T_e$. Then, it can be shown that as the effective delay spread increases, $\sigma$ and $|\rho|$ decrease whereas $\sigma^2$ increases. The desired signal power loss and interference power enhancement lead to the reduction of $\gamma$. Thus, the probability of error increases with the effective delay spread. Note that the hop duration of an FFH system $T/L$ is $L$ times smaller than that of an MCFH system $T$. This means that the effective delay spread for an FFH system is $L$ times larger than that of an MCFH system, when the delay spread is given. This is the reason that the MCFH system is superior to the FFH system in frequency-selective fading channels.
the BER performance of FFH systems in a small extent, and that of MCFH systems in a large extent. In FFH systems, the effective delay spread increases with diversity order. Due to an increase in the effective delay spread, the overall performance enhancement is reduced with an increase of diversity order. Whereas, in MCFH systems, the effective delay spread does not change with diversity order. Hence, diversity gain is greater for MCFH systems than for FFH systems.

It is well known that in static channels, the optimum values of $h$ for correlator-based noncoherent detection of BFSK signals are integer values to satisfy the orthogonality condition, if the multiple access interference is not considered [3][7]. In fading channels, however, orthogonality is not maintained due to frequency-selective fading and fast fading. Thus, the optimum value will vary with a channel condition. Figure 8 shows the BER performance of an MCFH system for various values of $h$ between 0.4 and 1.6, when $L = 3$, $f_p T = 0.01$, and $E_s/N_0 = 20 dB$. This range is a typical $h$ range in practical systems. For the given range of $h$ and maximum delay spread, it can be shown that the optimum value of $h$ increases as the delay spread increases. It can be explained as follows. As the delay spread increases, the interference power increases while the desired signal power decreases. The interference power may be reduced by increasing $h$. Note that the effect of delay spread should be considered in determining the optimum $h$.

V. CONCLUSIONS

The BER performance of FFH and MCFH systems in frequency-selective Rayleigh fading channels are presented and compared. The optimum combining rule is developed. The expressions for the probability of error are derived and evaluated for various channel conditions. It is found that frequency-selective Rayleigh fading severely degrades the performance of FHSS systems. MCFH systems are found to outperform FFH systems in frequency-selective fading environments. On the other hand, it is found that FFH systems are superior to MCFH systems in fast fading environments. Diversity is found to improve the performance of both MCFH and FFH systems. However, the diversity gain is greater for MCFH systems than for FFH systems. It is also shown that the optimum value of $h$ increases as the delay spread increases.

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