Use of Multiple Antennas for DS/CDMA Code Acquisition

Oh-Soon Shin and Kwang Bok (Ed) Lee
School of Electrical Engineering
Seoul National University
Kwanak P.O. Box 34, Seoul 151-742, Korea

Abstract—The effective use of multiple antennas is investigated for code acquisition. A generalized acquisition scheme is proposed for DS/CDMA systems with multiple antennas. The proposed scheme provides means of a trade-off between two important factors determining the mean acquisition time, which are referred to as combining gain and search time, respectively. The performance of the proposed acquisition scheme is analyzed in frequency-selective Rayleigh fading channels. In the analysis, the effects of spatial fading correlations on acquisition performance are considered. The mean acquisition time performance is evaluated for various environments, and the configuration of multiple antennas is investigated to reduce the mean acquisition time. The effects of spatial correlation on mean acquisition time are also investigated. The mean acquisition time is found to decrease significantly as the number of antenna elements increases. It is found that various parameters such as the number of antenna elements, operating signal-to-interference ratio (SIR), and the number of resolvable paths should be considered in determining configuration of multiple antennas to reduce mean acquisition time.

I. INTRODUCTION

Recently, the use of multiple antennas in direct-sequence code-division multiple-access (DS/CDMA) systems has come to receive considerable attention in mobile radio communications. Multiple antennas, often called as antenna arrays, with a spatial processing can enhance the desired signal and can suppress interfering signals, thereby improving performance and increasing the capacity of wireless systems [1]-[4]. A number of spatial processing techniques have been developed, which can be classified into beamforming or diversity combining techniques [2]. In DS/CDMA systems, however, the attractive features of these techniques can be exploited only after code timing is acquired. Hence, developments of effective code acquisition schemes in multiple antenna systems are crucial for the successful deployment of multiple antennas.

The code acquisition problem has been extensively investigated in a DS/CDMA system with a single antenna [5],[6]. Recently, there have been a few works dealing with the acquisition problem for multiple antenna systems. In [7], the maximum likelihood procedures for estimating the received code phase have been addressed in a static channel. However, these acquisition schemes are not suitable for systems with long code length because of the large amount of computation. An extension of the conventional noncoherent acquisition scheme to multiple antenna systems has been presented in [8]. In this scheme, the received signals at multiple antennas are combined noncoherently to achieve a diversity gain as well as an antenna gain, where the antenna gain is defined as an increase in the signal-to-interference ratio (SIR). Performance analysis has been conducted under the assumption that the received signals at multiple antennas experience uncorrelated fading. Results in [8] have shown significant acquisition performance improvement of a multiple antenna system compared to a single antenna system in uncorrelated fading environments.

In this paper, the effective use of multiple antennas is investigated for DS/CDMA systems with multiple antennas. We propose a generalized acquisition scheme, which includes the scheme in [8] as a special case. The proposed acquisition scheme provides means of a trade-off between two important factors that determines the mean acquisition time: combining gain and search time, where the combining gain represents diversity gain plus antenna gain of a noncoherent combining, and the search time is the time required to determine an in-phase cell in the first dwell. The performance of the proposed scheme is analyzed in frequency-selective Rayleigh fading channels. The mean acquisition time performance is evaluated in various environments, and the configuration of multiple antennas that reduces the mean acquisition time is investigated. The effects of various parameters such as the number of antenna elements, operating SIR, and the number of resolvable paths on antenna configuration are discussed. The effects of spatial correlation on mean acquisition time performance are also investigated.

The remainder of this paper is organized as follows. Section II describes the proposed acquisition scheme with multiple antennas. In Section III, performance analysis of the proposed acquisition scheme is presented in correlated Rayleigh fading channels. In Section IV, the mean acquisition time performance is evaluated in various environments, and the effects of antenna configuration and spatial correlation are discussed. Finally, conclusions are drawn in Section V.

II. PROPOSED ACQUISITION SCHEME

The acquisition scheme considered in this paper is a double-dwell noncoherent scheme with search and verification stages, as shown in Fig. 1. The receiving antennas are a uniform linear array of $L$ elements with spacing between adjacent antenna elements equal to $D$. It is assumed that one matched filter is employed at each antenna element to correlate the received signal samples with the local code sequence.

The proposed acquisition scheme provides means of a trade-off between the combining gain and search time. As depicted in Fig. 1, $L = M \times N$ antenna elements are partitioned into $N$ disjoint groups with $M$ antenna elements in each group. The $M$ matched filters in each group are used to

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get correlation results, which are associated with all phases in the entire uncertainty region. Correlation results associated with the same phase from $N$ groups are combined to form a decision variable, producing $N$ degrees of combining gain.

Specifically, the $n$th antenna group consists of antenna elements $M(n-1)+m$ ($m = 1, 2, \ldots, M$). The $M$ matched filters in each antenna group are loaded with $M$ different sets of coefficients as represented by \( 'MF_m' \) ($m = 1, 2, \ldots, M$) in Fig. 1. The coefficients of \( 'MF_m' \) \( \{c_{(m-1)A}, c_{(m-1)A+1}, \ldots, c_{(m-1)A+K-1} \} \) and those of \( 'MF_m+1' \) \( \{c_{mA}, c_{mA+1}, \ldots, c_{mA+K-1} \} \) have a phase difference of \( AT \equiv \frac{U}{M}\pi \), where \( \lfloor x \rfloor \) denotes the smallest integer equal to or greater than $x$. This allows the $M$ matched filters in each group to produce correlation results corresponding to $M$ different code phases simultaneously, which are separated by $AT$ in the uncertainty region. Hence, the correlation results for $U$ different code phases can be obtained during $AT$, resulting in a total $AT$ of search time. The decision variable for each code phase is obtained by combining $N$ matched filter outputs from $N$ different groups. The decision variable $S_{m}$ is constructed by combining $N$ outputs of the $m$th matched filter in each group, and it is expressed as

$$S_{m} = \sum_{n=1}^{N} |\hat{y}_{m+1}|, \quad m = 1, 2, \ldots, M.$$  

(1)

Note that $M$ and $N$ are design parameters that determine a trade-off between the combining gain and search time. The larger $N$ produces the greater combining gain and the longer search time. A special case of $N=L$ corresponds to the scheme in [8]. With an appropriate choice of $M$ and $N$, we can reduce the mean acquisition time compared to the scheme in [8]. Another point to be noted is that the antenna elements associated with matched filter outputs to be combined are separated by $MD$, which is the maximum uniform separation that can be achieved by $N$ elements chosen from $L$ linearly spaced antenna elements. Wider antenna separation provides a greater diversity effect, since the signals become less correlated with wider separation.

In the verification stage, the $L$ matched filters are loaded with coefficients corresponding to the phase selected in the search stage, and they are advanced by the same rate as the incoming code, performing correlations for the selected phase. The decision variable $V$ is constructed by combining $L$ matched filter outputs \( \{z_1, z_2, \ldots, z_L \} \) as in the search stage:

$$V = \sum_{f=1}^{L} |z_f|^2.$$

(2)

The decision variable is compared with the decision threshold $\eta$. If the decision variable exceeds the threshold, acquisition is declared and the tracking system is enabled. Otherwise, the acquisition system goes back to the search stage.

III. PERFORMANCE ANALYSIS

In this section, the performance of the acquisition scheme described in Section II is analyzed in frequency-selective Rayleigh fading channels with spatial correlations. The received signal model is given in Section III-A, and equations for the probabilities of detection and false alarm are derived in Section III-B. An expression for the mean acquisition time is presented in Section III-C.

A. Received Signal Model

It is assumed that a DS/CDMA signal is received without data modulation. The complex baseband equivalent of the received signal at the $\ell$th antenna element may be expressed as

$$r_{\ell}(t) = \sqrt{P} \sum_{p=1}^{L_p} \alpha_p(\ell;p;t) e^{j2\pi f_s(t-\tau_p)} + n_\ell(t).$$

(3)

where $P$ denotes the average total received signal power, $f_s$ is the frequency offset between the transmitter and receiver, $c(t)$ is the pseudo-noise (PN) code waveform with duration $T_c$, $\tau_p$ is the received code phase for the $p$th resolvable path, and $L_p$ is the number of resolvable paths. $n_\ell(t)$ is a complex additive white Gaussian noise (AWGN) process with one-sided power spectral density $N_0$, and it represents noise plus multiple-access interference.

In (3), the multipath fading channel for the $p$th resolvable path at the $\ell$th antenna element is denoted as $a_{\ell}(p);t$, which is a complex Gaussian random process. Assuming that each path experiences independent Rayleigh fading, the correlation function of $a_{\ell}(p);t$ may be expressed as

$$E[\alpha_\ell(p;l)\alpha_\ell^*(q;s)]=\Phi(p)\delta[p-q]\cdot J_0(2\pi f_p(t-s))\cdot C_p(k,\ell).$$

(4)

where $E[\cdot]$ denotes the statistical expectation, $\Phi(p)$ denotes the multipath intensity profile, $f_p$ is the Doppler spread, and $J_0(.)$ represents the zeroth-order Bessel function of the first kind. Assuming that the angle of arrival of the $p$th path signal is uniformly distributed over $[\phi_p-\Delta_\phi, \phi_p+\Delta_\phi]$, the spatial correlation $C_p(k,\ell)$ in (4) between the $p$th path signals at the $k$th and $\ell$th antenna elements is calculated as [3]
$$C_p(k, \ell) = \frac{1}{2\Delta_p} \int_{-\pi}^{\pi} e^{\frac{2\pi Dl_{k-\ell} \sin \theta}{\lambda}} d\theta$$

(5)

where $\lambda$ is the carrier wavelength. In (4), note that the power of $a_{s}(p; t)$ is normalized so that \( \sum_{p=1}^{L_p} E[|a_{s}(p; t)|^2] = 1 \).

B. Probabilities of Detection and False Alarm

The receiver is assumed to be chip-synchronized to the received signal, and the tap spacing of matched filters is set to the chip duration $T_c$. Thus, the number of code phase uncertainties $U$ is equal to the PN code length. Acquisition is assumed if any of the $L_p$ resolvable paths is acquired, which implies that there exist $L_p$ in-phase cells. To calculate the probabilities of detection and false alarm, the probability density function (pdf) and cumulative distribution function (cdf) of the decision variables $S_{0}$ and $V$ are required to be found for a given hypothesis. From (1) and (2), note that decision variables are noncoherent combinations of matched filter outputs. We first derive the covariance matrices of the matched filter outputs to be combined, and then derive the pdf and cdf equations, the probabilities of detection and false alarm are calculated.

The matched filter output $z$, at the $l$th antenna element may be expressed as

$$z_l = \frac{1}{\sqrt{K_Tc}} \int_{0}^{K_Tc} r_l(t)e^{(t-\xi)}dt$$

(6)

where $K_Tc$ is the correlation interval, and $\xi$ is the local code phase of the matched filter. Under the hypothesis corresponding to $\xi = \xi_p$, which is denoted as $H^r_{\xi_p}$, the local code phase is aligned to the $p$th path signal. In this case, the matched filter output in (6) may be expressed as

$$z_l = \frac{F_{\xi_p}}{\sqrt{K_Tc}} \int_{0}^{K_Tc} a_{s}(p; t)e^{2\pi f_{r},(t-\xi)}dt + \frac{1}{\sqrt{K_Tc}} \int_{0}^{K_Tc} n_l(t)e^{(t-\xi)}dt$$

under $H^r_{\xi_p}$. (7)

Using (4) and (7), the correlation $R^r_{l}(k, \ell)$ between $z_l$ and $z_{\ell}$, which are obtained at the same time from matched filters with the same coefficients, under the hypothesis $H^r_{\xi_p}$ may be calculated as

$$R^r_{l}(k, \ell) \triangleq E[z_l z_{\ell}^*]$$

$$= \frac{P}{K_Tc} \int_{0}^{K_Tc} \int_{0}^{K_Tc} E[a_{s}(p; t)a_{s}^*(p, s)] e^{j2\pi f_{r}(t-s)} dt ds$$

$$+ \frac{1}{K_Tc} \int_{0}^{K_Tc} \int_{0}^{K_Tc} E[n_l(t)n_{\ell}^*(s)] e^{j(\xi-t)\eta}(\xi-s) ds$$

$$= PT\Phi(p)C_p(k, \ell) \left[ 1 + \frac{2K_Tc}{\pi} J_0(2\pi f_{r}T_c) \cos(2\pi f_{r}T_c)(1-m/K) \right]$$

$$+ N_0\delta[k - \ell]$$

(8)

where $E[n_l(t)n_{\ell}^*(s)] = N_0\delta[k - \ell]\delta[t - s]$, and $\delta[n]$ denotes the delta function, defined as 1 for $n = 0$ and 0 otherwise. Note that $PT/N_0$ is defined as SIR/chip. The matched filter output under the hypothesis corresponding to $\xi \neq \xi_p$ for $p = 1, 2, \ldots, L_p$, which is denoted as $H_0$, may be expressed as

$$z_{\ell} = \frac{1}{\sqrt{K_Tc}} \int_{0}^{K_Tc} n_{\ell}(t)e^{(t-\xi)}dt,$$ 

and the correlation $R_0(k, \ell)$ between $z_l$ and $z_{\ell}$, which are obtained at the same time from matched filters with the same coefficients, under the hypothesis $H_0$ is calculated as

$$R_0(k, \ell) \triangleq E[z_l z_{\ell}^*|H_0] = N_0\delta[k - \ell].$$

(9)

Since the distribution of $S_{0}$ in (1) is independent of $m$ for a given hypothesis, we omit the subscript $m$ hereafter. Since $z_{\ell}$'s ($l = 1, 2, \ldots, L$) are complex Gaussian random variables, the pdf and cdf of $S$ can be calculated using the partial fraction expansion of the characteristic function [9]. Under the hypothesis $H^r_{\xi_p}$, the pdf and cdf equations can be calculated as [9]

$$f_{s}(r|H^r_{\xi_p}) = \sum_{n=1}^{N} a_{s}^*(n) e^{-i\gamma_{\xi_p}(n)}$$

(11)

$$F_{s}(r|H^r_{\xi_p}) = \sum_{n=1}^{N} a_{s}^*(n)(1-e^{-i\gamma_{\xi_p}(n)})$$

(12)

where $\gamma_{\xi_p}(n)$'s are the eigenvalues of the covariance matrix $R^r_{\xi_p}$, whose $(k, \ell)$ element $R^r_{k,\ell}(k, \ell)$ is given in (8), and $a_{s}^*(n) = \prod_{i=0}^{L-1} \frac{1}{2}(1-\gamma_{\xi_p}(i)/\gamma_{\xi_p}(n))$. Note that all of the eigenvalues are assumed to be distinct in (11) and (12). When the same eigenvalues exist, the pdf and cdf equations can also be derived. Under the hypothesis $H_0$, the pdf and cdf equations are calculated as [9]

$$f_{s}(r|H_0) = \frac{r^{N-1}}{N_0^N}(N!1)^{-e^{-r/N_0}}$$

(13)

$$F_{s}(r|H_0) = 1-e^{-r/N_0} \sum_{n=1}^{N} \left( r/N_0 \right)^n$$

(14)

Note that the pdf and cdf equations of $V$, denoted as $f_{v}(r|H^r_{\xi_p})$ and $F_{v}(r|H^r_{\xi_p})$ hereafter, are the same as those of $S$ with the substitutions of $M$ and $N$ by 1 and $L$, respectively.

Using the pdf and cdf equations, the probability of detection $P_{D0}(p)$ for the $p$th resolvable path and that of false alarm $P_{F1}$ in the search stage may be calculated as

$$P_{D0}(p) = \int_{0}^{\infty} f_{s}(r|H^r_{\xi_p}) \left( \prod_{i=0}^{L-1} F_{v}(r|H_{0}) \right) (F_{v}(r|H_{0}))^{L-p} dr$$

$$P_{F1} = \frac{1}{L} \sum_{p=1}^{L} P_{D0}(p).$$

(15)

Similarly, the probability of detection $P_{D2}(p)$ for the $p$th resolvable path and that of false alarm $P_{F2}$ in the verification stage are calculated as

$$P_{D2}(p) = 1-F_{v}(\eta|H^r_{\xi_p})$$

$$P_{F2} = 1-F_{v}(\eta|H_0)$$

(16)

where $\eta$ is the decision threshold.

C. Mean Acquisition Time

The mean acquisition time can be calculated using the flow graph method. The transfer function $H(z)$ to the acquisition state can be calculated as [10]
the time required to collect a decision in (5) between adjacent antenna elements varies with the number of antenna groups \( N \), the number of resolvable paths \( L_p \), and frequency offset \( f \), which are normalized by the chip rate \( 1/T_c \). In (20), \( P_D \) and \( P_F \) denote the overall probability of detection and that of false alarm, respectively. Using (17)-(20), the mean acquisition time \( E[T_{ACQ}] \) can be calculated as

\[
E[T_{ACQ}] = \frac{dH(z)}{dz} \bigg|_{z=1} = \frac{\left(U/M\right) - K + JP_F}{P_D}.
\]

IV. NUMERICAL RESULTS

In this section, the performance of the proposed acquisition scheme analyzed in Section III is evaluated. The code period \( U \) and the correlation length \( L \) are set to 1024 and 256, respectively, and the penalty time \( J \) is assumed to be 10\(^7\) chips. The normalized Doppler spread \( f_D/T_c \) and frequency offset \( f_j/T_c \), which are normalized by the chip rate \( 1/T_c \), is set to 10\(^{-7}\) and 0, respectively. The multipath intensity profile \( \Phi(p) \) is assumed to be uniform, i.e., \( \Phi(p) = 1/L_p \) for \( p = 1, 2, \ldots, L_p \). The decision threshold \( \eta \) in the verification stage is numerically determined to minimize the mean acquisition time for each condition.

Fig. 2 shows how the magnitude of the correlation coefficient in (5) between adjacent antenna elements varies with the antenna spacing \( D \), angular spread \( 2\Delta \), and mean angle of arrival \( \phi \). The magnitude of correlation coefficient is observed to decrease as the antenna spacing and/or angular spread increases. Also, the correlation is higher for \( \phi = 90^\circ \) than for \( \phi = 0 \) (broadside). Note that the correlation is higher than 0.9 at \( D = \lambda/2 \), when \( \Delta \) is smaller than \( 10^\circ \), which is typical at a base station.

The effects of antenna spacing and angular spread on mean acquisition time are depicted in Fig. 3, when the number of antenna elements \( L = 4 \), the number of antenna groups \( N = 4 \), the number of resolvable paths \( L_p = 1 \), and the mean angle of arrival \( \phi = 0 \). The mean acquisition time is calculated using (15), (16), (20), and (21). For SIR/chip values equal to or greater than –20 dB, the mean acquisition time is found to decrease as the antenna spacing \( D \) and/or angular spread \( 2\Delta \) increases, or as the spatial correlation decreases. The reason for this is that diversity gains achieved in the noncoherent combining in (1) and (2) are greater for a smaller correlation value. When \( D \) is as large as 10\(\lambda\), the mean acquisition time is almost indistinguishable for different values of angular spread, since the wide antenna spacing makes the correlation among antenna elements sufficiently small. On the other hand, when \( \Delta \) exceeds \( 45^\circ \), almost no performance improvement is achieved by increasing the antenna spacing wider than \( \lambda/2 \). This is because the magnitude of the correlation coefficient between adjacent antenna elements is lower than 0.4 even at \( D = \lambda/2 \) and \( \Delta = 45^\circ \) in Fig. 2. The mean acquisition time for \( \Delta = 3^\circ \) is only 1.5 times greater than that for \( \Delta \) greater than \( 45^\circ \), when \( D = \lambda/2 \) and SIR/chip = –15 dB. In Fig. 3, it is interesting to see that the mean acquisition time increases as antenna spacing and/or angular spread increases for SIR/chip = –25 dB, which is an opposite trend to higher SIR/chip values. At \( D = \lambda/2 \), for example, the mean acquisition time for \( \Delta \) greater than \( 45^\circ \) is 1.5 times greater than that for \( \Delta = 3^\circ \). This implies that the operating SIR should be higher than some threshold for the diversity effect to be beneficial to acquisition performance.

Figs. 4 and 5 show how the mean acquisition time varies with the number of antenna groups \( N \), for the number of antenna elements \( L = 4 \) and \( L = 8 \), respectively. The antenna spacing \( D \) is set to \( \lambda/2 \). Both a frequency-nonselective fading channel with \( L_p = 1 \) and a frequency-selective fading channel with \( L_p = 3 \) are considered. For the frequency-nonselective fading channel, the mean angle of arrival \( \phi \) and the angular spread \( 2\Delta \) are set to 0 and \( 6^\circ \), respectively. For the frequency-
selective fading channel, the mean angles of arrivals \((\phi_1, \phi_2, \phi_3)\) and the angular spreads \((2\Delta_1, 2\Delta_2, 2\Delta_3)\) are, respectively, assumed to be \((-45^\circ, 0^\circ, 45^\circ)\) and \((6^\circ, 10^\circ, 20^\circ)\). In Figs. 4 and 5, it is observed that the larger \(N\) provides the shorter mean acquisition time at low SIR values, while the smaller \(N\) provides the shorter mean acquisition time at high SIR values for both frequency-nonselective and frequency-selective fading channels. This can be explained as follows. As \(N\) increases, the decision variables become more reliable due to increased combining gain, while the search time \([U/M/T]\) increases. The effect of the former on mean acquisition time is more significant at low SIR values, while that of the latter becomes dominant and the mean acquisition time becomes proportional to \(N\). These results indicate that the multiple antennas should be configured with an appropriate choice of \(N\) to reduce mean acquisition time. The value of \(N\) associated with the best choice becomes smaller for higher operating SIR, for smaller number of resolvable paths, and for larger number of antenna elements.

V. CONCLUSIONS

The effective use of multiple antennas has been investigated for code acquisition in DS/CDMA systems. We have proposed a generalized acquisition scheme that provides means of a trade-off between the combining gain and search time, by introducing a grouping of multiple antennas. The performance of the proposed acquisition scheme has been analyzed in frequency-selective Rayleigh fading channels with consideration of spatial fading correlations. The mean acquisition time performance has been evaluated for various environments, and effects of the spatial correlation and configuration of multiple antennas on mean acquisition time have been investigated. It has been shown that low spatial correlation is advantageous to reducing the mean acquisition time at high SIR values, while high spatial correlation is advantageous at low SIR values. An increase in the number of antenna elements has been found to reduce the mean acquisition time significantly. It is found that various parameters such as the number of antenna elements, operating SIR, and the number of resolvable paths should be considered in determining configuration of multiple antennas to reduce mean acquisition time.

REFERENCES