Acquisition for DS/CDMA Systems with Multiple Antennas in Frequency-Selective Fading Channels

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Abstract—A code acquisition scheme is proposed for DS/CDMA systems with multiple antennas. The proposed scheme employs a new search scheme, referred to as sparse search, to utilize multipath signals in reducing mean acquisition time over frequency-selective fading channels. The mean acquisition time performance of the proposed scheme is analyzed and evaluated in frequency-selective Rayleigh fading channels. The performance of the proposed scheme is compared with that of the conventional one. Numerical results show that the proposed acquisition scheme outperforms the conventional one, and that the performance improvement becomes greater as the number of resolvable paths increases. The effects of configuration of multiple antennas and receiver complexity on mean acquisition time are also investigated.

I. INTRODUCTION

Recently, the use of multiple antennas in direct-sequence code-division multiple-access (DS/CDMA) systems has come to receive considerable attention in mobile radio communications. Multiple antennas, often called as antenna arrays, with a spatial processing can enhance the desired signal and can suppress interfering signals, thereby improving performance and increasing the capacity of wireless systems [1], [2]. A number of spatial processing techniques have been developed to exploit the attractive features of multiple antennas [2]. In DS/CDMA systems, however, these techniques are useful only after code timing is acquired. The effective use of multiple antennas can also improve the performance of code acquisition [3], [4]. A simple extension of the classical acquisition scheme for single antenna systems to multiple antenna systems has been proposed in [4], and the performance has been analyzed in fading environments.

In frequency-selective fading channels, there exist a number of resolvable paths. From the viewpoint of acquisition, the existence of multipaths implies that there exist more than one in-phase cells [5]. In next generation DS/CDMA systems, wide bandwidths are employed to provide high data rate services, resulting in an increase in the number of resolvable paths. Therefore, effects of multipaths on acquisition performance may become more important in next generation systems. There have been some attempts to utilize multipath signals in improving acquisition performance. One of these attempts is presented in [5], where an acquisition scheme based on the nonconsecutive search has been proposed. The nonconsecutive search is employed to decrease the search time by testing cells with step size greater than one chip. It has been shown that the nonconsecutive search provides significant reduction of mean acquisition time in frequency-selective fading channels.

In this paper, we propose an effective acquisition scheme for DS/CDMA systems with multiple antennas over frequency-selective fading channels. In the proposed acquisition scheme, a new search strategy, referred to as a sparse search, is employed to utilize multipath signals in improving acquisition performance. The sparse search can be viewed as a modification of the conventional parallel search. In the conventional parallel search, a decision is made based on decision variables corresponding to all the test cells [4]. In the sparse search, however, decision variables associated with only a subset of test cells are concerned to decide which cell is an in-phase cell. The number of resolvable paths determines the number of cells in a subset, or the sparseness of code phases in a subset. As in the nonconsecutive search [5], the sparse search can be implemented by updating the phase of local code generator by more than one chips. This process makes each subset of cells contain an in-phase cell. The benefits of sparse search arise from the reduction of search time, which is defined as the time required for collecting decision variables to make a decision. The mean acquisition time performance of the proposed acquisition scheme is analyzed in frequency-selective Rayleigh fading channels. The performance is evaluated and compared with that of the conventional scheme. The effects of configuration of multiple antennas and receiver complexity on mean acquisition time performance of the proposed scheme are also investigated.

The remainder of this paper is organized as follows. Section II describes the proposed acquisition scheme. In Section III, the performance of the proposed scheme is analyzed in frequency-selective Rayleigh fading channels. In Section IV, the performance of the proposed scheme is evaluated and compared with each other. Finally, conclusions are drawn in Section V.

II. PROPOSED ACQUISITION SCHEME

The proposed acquisition scheme is a double-dwell noncoherent scheme with search and verification stages as depicted in Fig. 1. The basic framework of this scheme is the same as the scheme proposed in [4], except that sparse search and
active correlators are employed instead of parallel search and matched filters. The receiving antennas are a uniform linear array of $L$ elements with spacing between adjacent antennas equal to $D$. At each antenna, $\Omega$ correlators with correlation interval of $K$ chips are employed to correlate the received signal with a local code as shown in Fig. 2. As proposed in [4], $L (=M \times N)$ antenna elements are partitioned into $N$ disjoint groups with $M$ antenna elements in each group. The $M\Omega$ correlators in each group are used to get correlation results for $M\Omega$ different phases simultaneously. Correlation results associated with the same phase from $N$ groups are combined to form a decision variable, producing $N$ degrees of combining gain. Specifically, the $n$th antenna group consists of antenna elements $M(n-1)+m \ (m=1, 2, \ldots, M)$. The $M\Omega$ local code generators of correlators have phase difference of $\Delta \Omega \equiv \lceil U/M\Omega \rceil T$, where $U$ is the number of code phase uncertainties, $T$ is the search step size of correlators, and $\lceil x \rceil$ denotes the smallest integer equal to or greater than $x$. This allows $M\Omega$ correlators in each group to produce correlation results corresponding to $M\Omega$ different code phases at the same time, which are separated by $\Delta T$ in the uncertainty region. The decision variable for each code phase is obtained by combining $N$ correlator outputs from $N$ different groups. $M$ and $N$ are design parameters that provides a trade-off between combining gain and search time. The larger $N$ produces the greater combining gain and the longer search time [4]. These parameters should be chosen to reduce the mean acquisition time. Another design parameter $\Omega$ affects the search time and complexity of the receiver. The larger $\Omega$ provides the shorter search time at the expense of the increased complexity of the receiver.

In order to exploit the presence of more than one in-phase cells ($H_I$ cells) in frequency-selective fading channels, sparse search strategy is proposed for the search stage. In the conventional parallel search, decision variables corresponding to $U$ test cells are collected to decide which is an $H_I$ cell. In the sparse search, however, decision variables associated with a subset of $U$ test cells are concerned to make a decision. The test cells in each subset have a spacing of $L_p$ chips in the code phase, where $L_p$ denotes the number of resolvable paths, and it is assumed to be known to the receiver. The total number of subsets is $L_p$, and the $\mu$th subset contains $\Theta_H$ test cells, in which one cell is an $H_I$ cell corresponding to the $\mu$th resolvable path. The sparse search can be implemented by advancing the phase of local code generators by $L_p$ chips, as depicted in Fig. 1. This process makes each subset of cells contain an in-phase cell. The benefits of sparse search compared to parallel search come from the reduction of search time required to make a decision from $U \cdot K T_c$ to $\Theta_H K T_c$. The search procedure and benefits of sparse search may be described using the circular state diagram in Fig. 3. In this figure, the state $\mu$ denotes the state corresponding to the $\mu$th subset containing $\Theta_H$ cells, in which one cell is an $H_I$ cell corresponding to the $\mu$th resolvable path. Acquisition can take place at any $L_p$ states.

In each state, the code phase associated with the largest decision variable is tentatively assumed as an $H_I$ cell, and the verification stage is activated. In the verification stage, the phase of local code associated with one of $\Omega$ correlators in each antenna element is adjusted to the phase selected in the search stage, and correlations are performed on the phase. The decision variable is constructed by combining $L$ correlation results from $L$ different antennas, and it is compared with the decision threshold $\gamma$. If the decision variable exceeds the threshold, acquisition is declared and tracking system is enabled. Otherwise, the acquisition system goes back to the search stage. In case of a false alarm on a state, acquisition is resumed in the next state after $J$ chips of penalty time.
In this section, the performance of the acquisition scheme described in Section II is analyzed in frequency-selective Rayleigh fading channels. The received signal model is given in Section III-A. The derivations of the probabilities of detection, miss-detection, and false alarm can be conducted in the same manner as in [4], and they are briefly summarized in Section III-B. In Section III-C, an expression for the mean acquisition time is derived using the circular state diagram in Fig. 3.

### A. Received Signal Model

It is assumed that a DS/CDMA signal is received without data modulation. The complex baseband equivalent of the received signal at the \( \ell \)th antenna element may be expressed as

\[
r(\ell,t) = \sqrt{P} \sum_{\mu} a(\mu,t) e^{j2\pi\mu/c (t - \tau)} + n(\ell,t)
\]

where \( P \) denotes the average total received signal power, \( f_o \) is the frequency offset between the transmitter and receiver, \( c(t) \) is the pseudo-noise (PN) code waveform with duration \( T_c \), \( \tau \) is the received code phase for the \( \mu \)th resolvable path and the \( \ell \)th antenna element is denoted as \( a(\mu,t) \), which is a complex Gaussian random process. Assuming that each path experiences independent Rayleigh fading, the correlation function of \( a(\mu,t) \) may be expressed as [4]

\[
E[a(\mu,t)\bar{a}(\nu,s)] = \Phi(\mu)|\delta[\mu - \nu| \cdot J_0 (2\pi f_o (t - s)) \cdot C_\mu (k,\ell)
\]

where \( E[\cdot] \) denotes the statistical expectation, \( \Phi(\mu) \) denotes the multipath intensity profile, \( \delta[\cdot] \) denotes the delta function, defined as 1 for \( n = 0 \) and 0 otherwise, \( f_o \) is the Doppler spread, and \( J_0 (\cdot) \) represents the Bessel function of the first kind of order zero. Assuming that the angle of arrival of the \( \mu \)th path signal is uniformly distributed over \( [0,\pi|\Delta|, \phi+\Delta] \), the spatial correlation \( C_\mu (k,\ell) \) in (2) between the \( \mu \)th path signals at the \( k \)th and \( \ell \)th antenna elements is calculated as [4]

\[
C_\mu (k,\ell) = \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} e^{2\pi i (k - \ell) \sin \theta} d\theta \quad (3)
\]

where \( \lambda \) is the carrier wavelength. In (2), note that the power of \( a(\mu,t) \) is normalized so that \( \sum_{\mu=1}^{L} E|a(\mu,t)|^2 = 1 \).

### B. Probabilities of Detection, Miss-Detection, and False Alarm

The receiver is assumed to be chip-synchronized to the received signal, and the search step size \( T \) of each correlator is set to the chip duration \( T_c \). Thus, the number of code phase uncertainties \( U \) is equal to the PN code length. Acquisition is assumed to be completed if any of the \( L_p \) resolvable paths is acquired, which implies that there exist \( L_p \) in-phase cells. The probability density function (pdf) and cumulative distribution function (cdf) of the decision variables in the search and verification stages can be calculated using the characteristic function method, and detailed procedures are described in [4]. Assuming that these equations are given, we can calculate the probability of detection \( P_{d1}^\mu \), and that of false alarm \( P_{f1}^\mu \) in the search stage at the state \( \mu \) of Fig. 3 as

\[
P_{d1}^\mu = \int f_\mu (r[H_{1}\mu]) \cdot F_{\mu} (r[H_0]) \cdot \theta_{\mu}^{-1} dr, \quad P_{f1}^\mu = 1 - P_{d1}^\mu
\]

where \( f_\mu (r[H]) \) and \( F_{\mu} (r[H]) \) denotes the pdf and cdf equations of a decision variable \( \mu \) in the search stage for a given hypothesis. The hypothesis \( H_{1}\mu \) represents that the local code phase is aligned to the \( \mu \)th path signal, and \( H_0 \) represents that the local code phase is not aligned to any of the resolvable paths. In (4), \( \Theta_\mu \) denotes the number of test cells associated with the state \( \mu \), and it can be calculated as [5]

\[
\Theta_\mu = \left\lfloor \frac{1}{\lambda/L_p} \right\rfloor M \Omega, \quad 1 \leq \mu \leq \Lambda - \left\lfloor \frac{\lambda/L_p}{L_p} \right\rfloor L_p
\]

where \( \lfloor x \rfloor \) denotes the integer part of \( x \). The probability of detection \( P_{d1}^\mu \) for the \( \mu \)th resolvable path and that of false alarm \( P_{f1}^2 \) in the verification stage may be calculated as

\[
P_{d1}^\mu = 1 - F_{\mu} (\gamma[H_{1}\mu]), \quad P_{f2}^\mu = 1 - F_{\mu} (\gamma[H_0])
\]

where \( F_{\mu} (\gamma[H]) \) denotes the cdf equation of a decision variable \( V \) in the verification stage for a given hypothesis \( H \), and \( \gamma \) is the decision threshold. Finally, the overall probability of detection, that of false alarm, and that of miss-detection for the state \( \mu \) may be expressed as

\[
P_{d}^\mu = P_{d1}^\mu P_{f2}^\mu, \quad P_{f}^\mu = P_{f1}^\mu P_{f2}^\mu, \quad P_{MD}^\mu = 1 - P_{d}^\mu - P_{f}^\mu
\]
C. Mean Acquisition Time

The mean acquisition time of the proposed scheme may be calculated using the flow graph method [6]. In the circular state diagram in Fig. 3, the branch gains are expressed as

\[ H'_D(z) = P_{\mu}^{\text{ACQ}} \sum_{m=1}^{L_p} H_{\mu,m}^{(m)}(z), \quad \mu = 1, 2, \ldots, L_p, \]
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where \( H'_D(z) \) and \( H'_D(z) \) are, respectively, the gain of the branch connecting the state \( \mu \) and the ACQ, and that of the branch connecting the states \( \mu \) and \( ((\mu+1)) \), where \( ((x)) \) \( (x-1) \mod L_p+1 \).\( (8) \) (9)

Under the assumption that the initial state \( \mu \) is uniformly distributed over states \( 1, 2, \ldots, L_p \), the generating function \( H(z) \) is found as

\[ H(z) = \frac{1}{L_p} \sum_{\mu=1}^{L_p} H(z|\mu) = \frac{1}{L_p} \sum_{\mu=1}^{L_p} \sum_{j=1}^{L_p} H_{\mu,j}(z), \]

\[ H(z) = \frac{1}{L_p} \sum_{\mu=1}^{L_p} H(z|\mu) = \frac{1}{L_p} \sum_{\mu=1}^{L_p} \sum_{j=1}^{L_p} H_{\mu,j}(z). \]

The corresponding mean acquisition time may be calculated as [6]

\[ E[T_{\text{ACQ}}] = \frac{dH(z)}{dz} \bigg|_{z=1}, \]

which yields

\[ E[T_{\text{ACQ}}] = \frac{L_p}{L_p(1-H_{\text{ACQ}})} \left[ \sum_{\mu=1}^{L_p} \sum_{j=1}^{L_p} H'_D(1,\mu,j) \frac{L_p}{L_p(1-H_{\text{ACQ}})} \sum_{\mu=1}^{L_p} \sum_{j=1}^{L_p} H'_D(1,\mu,j) \right] \]

where

\[ H'_D(1,\mu,j) = H'_D(z) \prod_{m=1}^{L_p} H_{\mu,m}^{(m)}(1), \]

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\[ H'_D(1,\mu,j) = H'_D(z) \prod_{m=1}^{L_p} H_{\mu,m}^{(m)}(1). \]

IV. NUMERICAL RESULTS

In this section, the mean acquisition time performance of the proposed acquisition scheme is evaluated and compared with that of the conventional one. PN code length \( U \) and correlation length \( K \) are set to 1024 and 256, respectively, and the penalty time \( J \) is assumed to be 10\(^8\) chips. The normalized Doppler spread \( f_D T_c \) and frequency offset \( f_c T_c \) which are normalized by the chip rate \( 1/T_c \) are set to 10\(^8\) and 0, respectively. The spacing \( D \) between adjacent antenna elements is assumed to be \( \lambda/2 \), and the multipath intensity profile \( \Phi(\mu) \) is assumed to be uniform, i.e., \( \Phi(\mu) = 1/L_p \) for \( \mu = 1, 2, \ldots, L_p \). The decision threshold \( \gamma \) in the verification stage is numerically determined to minimize the mean acquisition time for each condition. Equations (15)-(20) are used to calculate the mean acquisition time of the proposed acquisition scheme. The conventional scheme refers to the scheme in [4], which employs parallel search instead of the sparse search of the proposed scheme. For the conventional scheme, the mean acquisition time is calculated using the equations in [4].

Figs. 4 and 5 show mean acquisition time performance of the proposed and conventional acquisition schemes, when the number of resolvable paths \( L_r \) is 5 and 10, respectively. In these figures, the number of antenna elements \( L \) and the number of correlators per antenna element \( \Omega \) are assumed to be 8 and 1, respectively. The angle of arrival \( \phi_0 \) and angular spread \( 2\Delta_\theta \) for each path are set to the values in Table I. For both schemes, the larger \( N \) provides the shorter mean acquisition time at low signal-to-interference ratio (SIR) values, while the smaller \( N \) provides the shorter mean acquisition time at high SIR values. This is because the effect of combining gain is more significant at low SIR values, while that of search time is more significant at high SIR values [4]. It is shown that the proposed acquisition scheme outperforms the conventional one for all cases, and that the performance improvement is greater at high SIR values. The reason for this is that the search time is \( L_r \) times smaller for the sparse search.

<table>
<thead>
<tr>
<th>( L_r = 5 )</th>
<th>( L_r = 10 )</th>
</tr>
</thead>
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<tr>
<td>( \mu )</td>
<td>( \phi_0 )</td>
</tr>
<tr>
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<td>-60(^\circ)</td>
</tr>
<tr>
<td>1</td>
<td>-30(^\circ)</td>
</tr>
<tr>
<td>3</td>
<td>0(^\circ)</td>
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<tr>
<td>4</td>
<td>45(^\circ)</td>
</tr>
<tr>
<td>5</td>
<td>90(^\circ)</td>
</tr>
<tr>
<td>8</td>
<td>45(^\circ)</td>
</tr>
</tbody>
</table>

Table I

ANGLE OF ARRIVAL AND ANGULAR SPREAD FOR EACH RESOLVABLE PATH
than for the parallel search. At a sufficiently high SIR, the probability of detection at any resolvable path is close to unity, whereas the probabilities of miss-detection and false alarm are small enough to be negligible. This makes the mean acquisition time of the proposed scheme become $L_p$ times smaller than that of the conventional one at high SIR. For example, when $M = 4$, $N = 2$, and SIR/chip = 0 dB in Fig. 4, the mean acquisition time of the proposed scheme is 14100 chips, which is close to $1/5$ of that of the conventional scheme, 65800 chips. At low SIR values, on the other hand, the effects of reduction in the search time becomes less significant due to non-negligible probability of miss-detection or false alarm. Consequently, the performance difference of the proposed and conventional schemes becomes small at low SIR values.

The effect of receiver complexity on mean acquisition time of the proposed scheme is depicted in Fig. 6, when $L = 8$, $L_p = 5$, $M = 4$, and $N = 2$. As expected, the mean acquisition time decreases as the number of correlators per antenna element $\Omega$ increases from 1 to 3. The performance improvement is more significant at high SIR values. This is because the search time $(\Theta/M\Omega)KT$, for each state is proportional to $1/\Omega$, and the reduction of search time is more effective to reducing the mean acquisition time at higher SIR values.

V. CONCLUSIONS

In this paper, an acquisition scheme has been proposed for DS/CDMA systems with multiple antennas. The proposed scheme employs sparse search to exploit multipath signals in reducing mean acquisition time over frequency-selective fading channels. The mean acquisition time performance of the proposed scheme has been analyzed and evaluated in frequency-selective Rayleigh fading channels. The performance of the proposed scheme has been compared with that of the conventional one, and the proposed scheme has been found to outperform the conventional one. The performance improvement becomes greater as the number of resolvable paths and operating SIR increase. The effects of configuration of multiple antennas and receiver complexity on mean acquisition time have also been investigated.

REFERENCES