USE OF DIFFERENTIALLY COHERENT COMBINING FOR DS/CDMA CODE ACQUISITION

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Abstract—The use of differentially coherent combining is proposed to improve the performance of a double-dwell acquisition system. The differentially coherent combining is employed to increase the reliability of a decision in the verification stage. The mean acquisition time performance of the acquisition scheme with the proposed combining scheme is analyzed in a Rayleigh fading channel, and compared with that of a conventional double-dwell acquisition scheme with noncoherent combining. It is shown that the proposed acquisition scheme outperforms the conventional one, and that the performance improvement increases as the frequency offset increases.

I. INTRODUCTION

In direct-sequence code division multiple access (DS/CDMA) systems, code synchronization is one of the most important parts. The code synchronization is usually achieved in two steps: acquisition for coarse alignment and tracking for fine alignment; the former is addressed in this paper. In the design of acquisition systems, an important goal is to reduce mean acquisition time, which is the average time that elapses prior to acquisition.

Various acquisition schemes have been investigated for rapid acquisition [1]. One approach to achieving fast acquisition is to use a double-dwell scheme rather than a single-dwell scheme [1][2]. The advantage of the double-dwell scheme results from the significant reduction of costly false alarms. A double-dwell scheme has two modes of operation: search mode and verification mode. The former is used to make a tentative decision on the received code phase, and the latter is used to verify the decision in the search mode. Therefore, it is desirable that the decisions in the verification mode should be reliable to avoid false alarms. A simple method to increase reliability is to increase the correlation interval of the correlator in the verification mode. However, the performance of this scheme is degraded severely in the presence of frequency offset and fading [2].

To overcome the problem in increasing the correlation interval, verification methods based on multiple observations have been investigated [2]-[4]. In these methods, a number of correlations are performed to obtain multiple observations for the cell under the test. The performance of this approach depends on how to decide whether the cell is the in-phase cell or not, using these observations. A majority logic type decision strategy has been investigated in [3], where each observation is independently tested, and the decision is made based on the number of observations passing the test. Although this scheme is more robust than the method of increasing the correlation interval, the performance of this scheme may be poor in the presence of severe frequency offset and/or fading. This is because in these environments, the result of each test becomes too unreliable for the majority logic decision to be effective. Another type of decision strategy is presented in [2] and [4], where the decision is made based on only one test using the decision variable formed by noncoherent combining of multiple observations. The noncoherent combining stage inherently increases the reliability of the decision variable. In [5], the performance of these two verification methods has been compared, and it has been found that the decision based on the noncoherent combining outperforms the majority logic type decision. We will refer to the acquisition scheme with noncoherent combining in the verification stage as the conventional double-dwell acquisition scheme, hereafter.

Recently, a differentially coherent combining scheme has been proposed in [6] as an effective method to combine multiple observations for slot synchronization in W-CDMA systems. In [6], it has been shown that this combining scheme is superior to the noncoherent combining scheme in the presence of frequency offset and fading. Motivated by this investigation, we propose employing the differentially coherent combining instead of the noncoherent combining in the verification mode for a double-dwell acquisition system. In the proposed acquisition system, differential processing is performed on multiple observations, and the outputs of the differential processing are combined to form a decision variable in the verification mode. The mean acquisition time performance of the proposed acquisition scheme is analyzed in a Rayleigh fading channel, and it will be shown that the proposed acquisition scheme outperforms the conventional double-dwell acquisition scheme.

are drawn in Section V.

II. PROPOSED ACQUISITION SYSTEM

The proposed acquisition system is a double-dwell system with search and verification modes, as depicted in Fig. 1. The code period $NT_c$ is discretized with a step size of $T_c$, the chip duration, resulting in $N$ cells of the uncertainty region. In the search mode, the decision variable $S$ corresponding to each test cell is collected using a matched filter correlator with a correlation interval of $M$ chips, and the code phase corresponding to the largest decision variable is tentatively assumed as the in-phase cell. In the verification mode, an active correlator with a correlation interval of $M$ chips performs a number of correlations for the cell selected in the search mode. To form a decision variable in the verification mode, $L$
successive observations, i.e., correlator outputs, denoted as $z_\ell$ ($\ell = 1, 2, \cdots, L$) in Fig. 1, are combined after differential processing, $z_{\ell+1}z_\ell$. Hence, the decision variable $V$ in the verification mode is expressed as

$$V = \left| \sum_{\ell=1}^L z_{\ell+1}z_\ell \right|.$$  

This decision variable is compared with a decision threshold $y$. If the decision variable exceeds the threshold, acquisition is declared and the tracking system is enabled. Otherwise, the acquisition system goes back to the search mode. In contrast to the proposed acquisition system, the decision variable for the conventional double-dwell acquisition system is formed as $V = \sum_{\ell=1}^L |z_\ell|^2$. The use of differential processing prior to combining reduces the effects of phase fluctuations due to fading and frequency offset, and allows the outputs of differential processing to be combined coherently without severe degradation [6].

### III. PERFORMANCE ANALYSIS

In this section, the performance of the acquisition system described in Section II is analyzed in a Rayleigh fading channel. The received signal model is described in Section III-A, and equations for the probabilities of detection and false alarm are derived in Section III-B. An expression for the mean acquisition time is presented in Section III-C.

#### A. Received Signal Model

It is assumed that a DS/CDMA signal is received from a pilot channel without data modulation, and the receiver is chip-synchronized to the received signal. The complex baseband equivalent of the received signal may be expressed as

$$r(t) = \sqrt{P} \cdot a(t)e^{j2\pi f_o \tau} c(t - \tau) + n(t)$$  

where $P$ is the average received signal power, $f_o$ is the frequency offset between transmitter and receiver, $c(t)$ is the PN code waveform, and $\tau$ is the received code phase. The multiplicative Rayleigh fading channel is denoted as $\alpha(t)$, which is a complex Gaussian random process with the autocorrelation function given as $E[\alpha(t)\alpha^*(t)] = J_0(2\pi f_o|t - \tau|)$ [7], where $J_k(\cdot)$ represents the $k$th-order Bessel function of the first kind, and $f_o$ is the Doppler spread. $n(t)$ is a complex additive white Gaussian noise (AWGN) process with one-sided power spectral density $N_0$, and it represents noise plus interference form other users.

#### B. Probabilities of Detection and False Alarm

1) Search Mode: As depicted in Fig. 1, the decision variable $S$ in the search mode is formed by squaring a matched filter output $y$: $S = |y|^2$. The probability density function (pdf) and cumulative distribution function (cdf) of $S$ under the hypothesis $H_k$ ($n = 0, 1$) can be written as [8]

$$f_S(r|H_n) = \frac{1}{\lambda_n} \exp\left(-\frac{r}{\lambda_n}\right)$$

$$F_S(r|H_n) = 1 - \exp\left(-\frac{r}{\lambda_n}\right)$$

where $H_k$ and $H_0$ denote the in-phase and out-of-phase cells, respectively. $\lambda_n$ denotes the conditional first moment of $S$, and it can be calculated as

$$\lambda_n \triangleq E\left[ S = |y|^2 | H_n \right]$$

$$\lambda_0 \triangleq E\left[ S = |y|^2 | H_0 \right]$$

$$\lambda_k \triangleq E\left[ S = |y|^2 | H_k \right]$$

The use of differential processing prior to combining reduces the effects of phase fluctuations due to fading and frequency offset, and allows the outputs of differential processing to be combined coherently without severe degradation [6].
of the PN sequence is assumed to be negligible compared to the noise plus multiple-access interference, and $\delta[n]$ denotes the delta function, defined as 1 for $n = 0$ and 0 otherwise. Note that $PT/N_0$ is defined as the signal-to-interference ratio per chip (SIR/chip). Using (3), the probability of detection $P_{D1}$ and that of false alarm $P_{F1}$ in the search mode may be calculated as

$$P_{D1} = \int \frac{f_z(r) |H_1|}{\sum f_z(r) |H_i|} \left\{ \frac{F_z(r | H_0)}{F_z(r | H_1)} \right\}^{N-1} \, dr, \quad P_{F1} = 1 - P_{D1}. \quad (5)$$

2) Verification Mode: The decision variable $V$ in the verification mode is constructed by combining $L$ observations, $z_{\ell}$ ($\ell = 1, 2, \ldots, L$), as described in (1). The pdf and cdf of $V$ under the hypothesis $H_i$ are expressed as

$$f_V(r | H_i) = \frac{r}{2\pi} \int_{-\pi}^{\pi} \rho \Phi_{V|V_0} (\rho \cos \phi, \rho \sin \phi | H_i) \cdot J_0 (r \rho) \, d\phi \, dr,$$

$$F_V(r | H_i) = \frac{r}{2\pi} \int_{-\pi}^{\pi} \Phi_{V|V_0} (\rho \cos \phi, \rho \sin \phi | H_i) \cdot J_0 (r \rho) \, d\phi \, dr, \quad (6)$$

where $V_I$ and $V_Q$ are, respectively, the real and imaginary parts of $\sum_{\ell=1}^{L} z_{\ell}^*$, i.e., $V_I = Z^I Q_a$, $V_Q = Z^Q Q_a$ [6]. $I$ denotes the $L$-dimensional identity matrix, $\Phi_{V|V_0} (\cdot | H_i)$ represents the joint characteristic function of $V_I$ and $V_Q$ under the hypothesis $H_i$, which is expressed as

$$\Phi_{V|V_0} (\mu, \nu | H_i) = \left[ \det(\mathbf{1} - j \mathbf{R}_n (\mu \mathbf{Q}_I + \nu \mathbf{Q}_Q)) \right]^{-1} \quad (7)$$

where $\mathbf{Q}_I$ and $\mathbf{Q}_Q$ are, respectively, the matrices for constructing $V_I$ and $V_Q$ from the observation vector $z = [z_1, z_2, \ldots, z_L]^T$, i.e., $V_I = Z^I Q_a$, $V_Q = Z^Q Q_a$ [6]. $I$ denotes the $L$-dimensional identity matrix, $\mathbf{R}_n \triangleq E[zz^H | H_n]$ is the covariance matrix of $z$ under the hypothesis $H_n$, and the $(k, \ell)$ element of $\mathbf{R}_n$ can be calculated in a similar manner as in (4):

$$(\mathbf{R}_n)_{k\ell} = E[z_k^* z_\ell] = PT e^{j2\pi(k-\ell)M/T_c} \sum_{m=-M}^{M-1} J_0 (2\pi((k-\ell)M + m) f_D T_c) \cdot \cdot e^{j2\pi mf_c} \cdot [1 - m/M] \cdot \delta[n-1] + N_0 \cdot \delta[k-\ell]. \quad (8)$$

From (6), the probability of detection $P_{D2}$ and that of false alarm $P_{F2}$ in the verification mode can be calculated as

$$P_{D2} = 1 - F_V (\gamma | H_1), \quad P_{F2} = 1 - F_V (\gamma | H_0) \quad (9)$$

where $\gamma$ is the detection threshold.

C. Mean Acquisition Time

The mean acquisition time can be calculated using the flow graph method in [3]. From the reduced state diagram depicted in Fig. 2, the transfer function $H(z)$ from the 'TEST' state to the 'ACQ' state is found as

$$H(z) = \frac{H_D(z)}{1 - H_M(z)} \quad (10)$$

where

$$H_D(z) = \frac{P_{D1} P_{D2} z^{(N+LM)T_c}}{1 - P_{D1} P_{D2} z^{(N+LM)T_c}}, \quad H_M(z) = (P_{D1}(1 - P_{D2}) + P_{F1}(1 - P_{F2})) z^{(N+LM)T_c} \quad (11)$$

In (11), note that $N_{T_c}$ and $L_{M_{T_c}}$ are, respectively, the time required to collect $N$ decision variables in the search mode, and the time required to obtain $L$ observations in the verification mode, and $K_{T_c}$ is the penalty time due to a false alarm. Using (10) and (11), the mean acquisition time can be calculated as

$$E[T_{ACQ}] = \frac{dH(z)}{dz} \bigg|_{z=1} = H_D(1) + H_M(1). \quad (12)$$

IV. NUMERICAL RESULTS

In this section, the mean acquisition time performance of the proposed acquisition scheme analyzed in Section III is evaluated and compared with that of the conventional double-dwell acquisition scheme. The performance of the conventional double-dwell scheme can easily be evaluated using (5), (9), (11) and (12) with the pdf and cdf equations replaced by those for the noncoherent combining in [6]. The code period $N$ and the correlation length $M$ are set to 1024 and 256, respectively, and the penalty time $K$ is assumed to be $10^5$ chips. The decision threshold $\gamma$ in the verification mode is numerically determined to minimize the mean acquisition time for each condition.

Fig. 3 shows the mean acquisition time performance when the number of observations $L$ is 5. It is shown that the proposed acquisition scheme outperforms the conventional one, especially for large frequency offset. This indicates that the differentially coherent combining scheme used in the proposed acquisition scheme provides greater combining gain than the noncoherent combining scheme. The effects of Doppler spread on the performance are observed to be negligible for both schemes in this figure. This is because the variation of the fading process is not significant during the second dwell time $2LM_{T_c}$ even when the normalized Doppler spread $f_D T_c$ is as large as $10^4$ corresponding to $f_D = 368.64$ Hz for the chip rate of 3.6864 MHz [9].

The effects of frequency offset are depicted in Fig. 4, when $L = 5$, $f_D T_c = 10^4$, and SIR/chip = $-12$ dB. Note that the frequency offset range $(3 \times 10^4, 3 \times 10^5)$ considered in this figure is reasonable in practical situations, since $f_D T_c = 3 \times 10^4$ corresponds to $f_D = 11$ kHz (5.5 ppm at 2 GHz carrier frequency) for the chip rate of 3.6864 MHz. Fig. 4 shows that the mean acquisition time performance is severely degraded by the frequency offset. The proposed
acquisition scheme outperforms the conventional one, and the performance difference becomes greater as the frequency offset increases. When $f_o T_c = 3 \times 10^{-3}$, the mean acquisition time of the proposed scheme is $2.4 \times 10^5$ chips, and that of the conventional scheme is $4.9 \times 10^5$ chips. In this case, the use of the proposed acquisition scheme produces about a 50% reduction in the mean acquisition time compared to the conventional scheme.

V. CONCLUSIONS

In this paper, the use of a differentially coherent combining scheme has been proposed for the verification mode of the double-dwell acquisition scheme. The mean acquisition time performance has been analyzed in a Rayleigh fading channel, and compared with that of the conventional double-dwell acquisition scheme. It has been shown that the proposed acquisition scheme outperforms the conventional one, and that the performance improvement becomes greater as the frequency offset increases.

REFERENCES


