Low-Complexity Unitary Preprocessing Scheme for Limited Feedback Multiuser MIMO Systems

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Abstract—In this paper, a practical downlink multiuser multiple-input multiple-output (MU-MIMO) scheme with low-rate feedback channel is considered. Firstly, we propose a preferred-beam feedback method, which effectively conveys information of user channel and inter-user interference. Secondly, we propose a new low-complexity method to compute unitary preprocessing matrix based on the feedback method. The proposed method reduces considerably computational complexity by utilizing SVD operation compared to the conventional unitary preprocessing scheme. Finally, for the case that the number of users exceeds the number of transmit antennas, we provide two simple user selection algorithms to exploit multiuser diversity. Numerical results show that the proposed low-complexity unitary preprocessing scheme achieves higher sum-rate than previous linear preprocessing schemes, especially at low SNR or with small number of active users.

Index Terms—Multiuser MIMO, MIMO broadcast channel, linear preprocessing, unitary preprocessing, limited-feedback

I. INTRODUCTION

Advances of multiuser multiple-input multiple output (MU-MIMO) techniques in wireless communication systems have provided theoretical evidences to attain significant sum-capacity increment in the last few years [1]. In MU-MIMO systems, Dirty paper coding (DPC) [2] is a well-known optimal strategy that achieves the capacity of the MIMO Gaussian broadcast channel. But, DPC is difficult to implement due to exponential increase for computational power according to the number of users. Therefore, recent research in this area has been focused on linear preprocessing schemes for the simplicity of precoding and decoding.

Nowadays, practical linear preprocessing schemes have been proposed such as Zero-forcing Beamforming (ZFBBF) [3], Block Diagonalization (BD) [4]. Per user unitary and rate control (PU2RC) [5] based on orthogonal beamforming. Moreover, it is shown that several linear preprocessing schemes perform asymptotically the same with DPC as the number of users goes to infinity [3]. Aforementioned ZFBBF, and BD, however, suffer from performance degradation when channel quantization errors lead to imperfect CSI at the transmitter (CSIT) due to low-rate feedback [6], [7]. Inaccurate CSIT from channel quantization errors result in remaining inter-user interference, which decreases sum-rate. Although quantization-based combining (QBC) proposed in [8] alleviates the degradation of ZFBBF by effectively utilizing multiple receive antennas, the improvement is limited by high SNR region, that is, interference-limited regime. On the other hand, PU2RC achieves better sum-rate performance in the low-rate feedback since the channel feedback, user scheduling, and preprocessing are jointly designed by its nature [5]. But, the major drawback of the PU2RC is that preprocessing vectors are constrained within the predefined codebook, which results in low preprocessing gain. To increases the preprocessing gain by relaxing the constraint, we have proposed an enhanced unitary preprocessing scheme [9]. However, the enhanced unitary preprocessing in [9] requires high computational complexity due to iterative feature of the algorithm and considers only the case that the number of transmit antenna is the same as the number of users.

In this paper, we propose a preferred-beam feedback method, which considers the effect of inter-user interference as well as user channel. We also propose a new low-complexity method to compute unitary preprocessing matrix for reducing computational complexity. Finally, for the case that the number of users exceeds the number of transmit antennas, we provide two simple user selection algorithms to exploit multiuser diversity. Numerical results show that the proposed unitary preprocessing scheme achieves higher sum-rate than previous linear preprocessing schemes, especially at low SNR or with small number of active users.

II. SYSTEM MODEL

We consider a single cell multiuser MIMO downlink system with $N_t$ antennas at the base station and $N_r$ antennas at each of $K$ users as depicted in Fig. 1. We use a simple channel model where the channel matrix from each user is the independent and identically distributed (i.i.d.) flat Rayleigh fading. The signal received by user-$k$ is given by

$$ y_k = r_k (H_{k} F x + n_k), \quad k = 1, 2, \cdots, K, $$

where $H_k \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix of user-$k$, $F \in \mathbb{C}^{N_t \times 1}$ is the unitary preprocessing matrix, $x_k \in \mathbb{C}^{N_t \times 1}$ is transmitted symbol vector, $r_k \in \mathbb{C}^{1 \times N_r}$ is the unit norm receive combining vector for user-$k$, and $n_k \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise vector at the receiver. Let each column vector in $F$ denote the unit norm transmit preprocessing vector where equal power is allocated. The base station has average power constraint $P$, $\text{tr}(F F^H) \leq P$ under the assumption of assuming that $E[xx^H] = I_{N_t}$. We assume perfect CSI at the user terminal, error-free, zero-delay feedback link and one signal stream per user. We use B-bit codebook where 2B quantized codeword vectors are generated by Grassmannian line packing (GLP) [10].

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III. PROPOSED LOW-COMPLEXITY UNITARY PREPROCESSING SCHEME

In this section, we first propose a preferred-beam feedback method, which effectively conveys information of user channel and inter-user interference at the feedback stage. We then propose a new low-complexity method to compute unitary preprocessing matrix based on SVD operation. Finally, for $K > N_t$ case, we provide two simple user selection algorithms.

A. Preferred-Beam Feedback Method

Linear minimum mean square error (LMMSE) combining is considered at each user. From [11], the user-$k$’s signal to interference plus noise ratio (SINR), denoted as $\eta_k$, is

$$\eta_k = \frac{1}{\left(\frac{\rho}{N_t} F^H H_k^H H_k F + I_{N_t}\right)^{-1}} - 1.$$  \hspace{1cm} (2)

where $\rho$ is the output signal to noise ratio (SNR) of the receiver. We have derived the following theorem on the of the unitary preprocessing scheme [12].

Theorem 1: If the preprocessing matrix is restricted to full rank unitary, the post value after LMMSE combining does not depend on other users’ preprocessing vectors.

Proof: from (2),

$$\eta_k = \frac{1}{\left(\frac{\rho}{N_t} F^H H_k^H H_k F + I_{N_t}\right)^{-1}} - 1$$

(a) $$= \frac{1}{\left(F^H \left(\frac{\rho}{N_t} H_k^H H_k + F I_{N_t} F^H\right)^{-1} F\right)} - 1$$

(b) $$= \frac{1}{\left(F^H \left(\frac{\rho}{N_t} H_k^H H_k + I_{N_t}\right)^{-1} F\right)} - 1$$

(c) $$= \frac{1}{f_k^H \left(\frac{\rho}{N_t} H_k^H H_k + I_{N_t}\right)^{-1} f_k} - 1,$$  \hspace{1cm} (3)

where (a) and (b) follow because $F^H F = FF^H = I_{N_t}$. In (c), we re-express (3) by using $k$th column vector $f_k$ of the preprocessing matrix.

Under the assumption of full rank unitary preprocessing matrix, the user-$k$ can calculate $\eta_k$ if the user-$k$ knows only its own preprocessing vector $f_k$. Consequently, each user needs to report its own preferred-beam vector to the base station. The preferred beam vector, denoted as $p_k$, is selected in the predefined codebook as

$$p_k = \arg \max_{f_k \in \{a_1, a_2, \ldots, a_B\}} \left\{ \frac{1}{f_k^H \left(\frac{\rho}{N_t} H_k^H H_k + I_{N_t}\right)^{-1} f_k} - 1 \right\}. $$  \hspace{1cm} (4)

User-$k$ sends the index of $p_k$ and $\eta_k$ to the base station. Note that the preferred beam vector indicates the vector whose direction maximizes $\eta_k$ among the codebook. Conventional channel feedback method which quantizes user channel by computing inner products between user channel and codeword in the codebook only considers user’s channel direction. Hence, inter-user interference from inaccurate CSI is not considered. On the contrary, a key advantage of the proposed preferred-beam feedback method is that it effectively conveys information of user channel and inter-user interference at the feedback stage.

B. SVD-based Unitary Preprocessing

For simplicity, we suppose $K = N_t$ here, and $K \geq N_t$ case will be dealt with in the next section with user selection algorithms.

The base station assembles feedback information, preferred beam vectors, from users. Aggregated preferred-beam matrix, denoted as $P$, is defined as

$$P = \left[ \begin{array}{cccc} p_1 & p_2 & \cdots & p_{N_t} \end{array} \right]. $$  \hspace{1cm} (5)

We propose a low-complexity method to construct unitary preprocessing matrix $F$ based on $P$. The proposed unitary preprocessing method uses SVD operation of matrix $P^H$.

$$P = \left[ \begin{array}{cccc} p_1 & p_2 & \cdots & p_{N_t} \end{array} \right] = U \sum V^H. $$  \hspace{1cm} (6)

And unitary preprocessing matrix $F$ is determined by using $U$ and $V$

$$F = UV^H = \left[ \begin{array}{cccc} f_1 & f_2 & \cdots & f_{N_t} \end{array} \right]. $$  \hspace{1cm} (7)

The determined unitary preprocessing matrix $F$ satisfies the following theorem.

Theorem 2: Unitary preprocessing matrix $F$ determined by (6), (7) maximizes sum of inner products between the pre-processing vectors and the preferred-beam vectors from users, i.e.,

$$F = \arg \max_{F \in SU(N_t)} \sum_{i=1}^{N_t} |p_i^H f_i| $$  \hspace{1cm} (8)

Proof: from (6)and Let $G$ be any arbitrary unitary matrix.

$$P^H G = V \sum T^H = V \sum T^H $$  \hspace{1cm} (9)
where \( \sum = \begin{bmatrix} \sigma_1 & \ldots & \sigma_{N_t} \end{bmatrix} \), \( V = \begin{bmatrix} v_1 & v_2 & \ldots & v_{N_t} \end{bmatrix} \), \( T = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{N_t} \end{bmatrix} \), \( G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{N_t} \end{bmatrix}^T \), \( T^H = U^H G \), \( T T^H = T^H T = I \), \( G^H G = GG^H = I \). So, \( \text{tr}(PH^G) \) can be re-expressed as

\[
\text{tr}(PH^G) = \sum_{i=1}^{N_t} |p_i^H g_i|^2 = \sum_{i=1}^{N_t} \sigma_i |v_i^{H}|^2 \leq \sum_{i=1}^{N_t} \sigma_i, \tag{10}
\]

where (a) follows because \( |v_i| = |t_i| = 1 \) and \( \sigma_i |v_i^{H}| \leq \sigma_i \). Equality in (a) is satisfied when \( v_i = t_i \). It means that \( V = T \). Thus, \( G = UT^H = UV^H \) completes the proof.

By the proposed SVD-based unitary preprocessing method, preprocessing vectors are determined for maximizing sum of inner-products with preferred-beam vectors.

### C. Two Simple User Selection Algorithms

Two simple user selection algorithms for \( K \geq N_t \) case are described in this section. The base station selects \( N_t \) users based on the feedback information, that is, \( p_k \) and \( \eta_k \).

1) **Max-SINR User Selection algorithm**: we propose a Max-SINR user selection (MSUS) algorithm. The proposed MSUS algorithm utilizes only \( \eta_k \) for user selection process. The algorithm details are summarized in **TABLE I**.

Since the proposed MSUS algorithm considers only SINR information, the complexity of the algorithm is extremely low.

2) **Modified Greedy User Selection Algorithm**: we propose a modified greedy user selection (MGUS) algorithm, which is based on the conventional greedy selection concept [13]. The major advantage of the MGUS algorithm against the MSUS algorithm is that the MGUS algorithm utilizes both \( p_k \) and \( \eta_k \) information, i.e., the MGUS algorithm selects \( N_t \) users with large SINR whose preferred-beam vectors are as orthogonal as possible. The MGUS algorithm is summarized in **TABLE II**.

Note that in the MGUS algorithm, we used preferred-beam vectors and SINR instead of channel direction and channel norm, respectively. Since the preferred-beam vector and SINR only convey partial information of user channel \( H \), the proposed MGUS algorithm based on \( p_k \) and \( \eta_k \) does not guarantee the best selected user set. However, the proposed MGUS algorithm provides high sum-rate performance with reasonable computational complexity as will be shown in simulation results.

### IV. Numerical Results

In this section, the sum-rate performance for the proposed low-complexity unitary preprocessing scheme is evaluated and compared with conventional linear preprocessing schemes. Simulation results are averaged over 1,000 independent channel realizations. In Fig.2, average sum-rate performance versus SNR is plotted for \( N_t = 4, N_r = 2, K = 4 \) and \( B = 4 \). The proposed unitary preprocessing scheme outperforms other enhanced unitary beamforming [9], ZFBF-QBC [8] and PU2RC [5] at all SNR region. Since unitary preprocessing matrix of PU2RC is restricted within the predefined codebook, PU2RC shows low sum-rate performance due to small preprocessing gain. On the contrary, the proposed low-complexity unitary preprocessing scheme exploits large preprocessing gain based on the unconstrained unitary preprocessing, which makes the proposed unitary preprocessing scheme achieve higher sum-rate than PU2RC. ZFBF-QBC and Enhanced unitary beamforming suffer from quantization error in low-rate feedback channel. In addition, inter-user interference from quantization errors cannot be controlled at preprocessing because interference information is not conveyed in the feedback. The proposed unitary preprocessing scheme considers the effect of inter-user interference as well as user channel in the feedback stage. Therefore, the proposed low-complexity unitary preprocessing scheme performs effective preprocessing based on both user channel and interference information.

**TABLE II**

**MAX-SINR USER SELECTION (MSUS) ALGORITHM**

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Initialization:</th>
</tr>
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<tbody>
<tr>
<td>( n = 1 )</td>
<td>( T_1, \ldots, T_{2B} (T_i = { k</td>
</tr>
<tr>
<td>( C = { c_1, c_2, \ldots, c_B } )</td>
<td>( U \supseteq T_i {</td>
</tr>
<tr>
<td>( S = \emptyset )</td>
<td>( S = S \cup { \pi } )</td>
</tr>
</tbody>
</table>

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<tr>
<th>Step 2</th>
<th>Iterative user selection:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi(n) = \arg \max_{\pi_k} | \eta_k | )</td>
<td>( S \leftarrow S \cup { \pi } )</td>
</tr>
<tr>
<td>( n \leftarrow n + 1 )</td>
<td>( U \leftarrow U \setminus { \pi } )</td>
</tr>
</tbody>
</table>

| Step 3 | Algorithm termination: If \( |S| < N_t \), then go to step 2. |

**TABLE I**

**MAX-SINR USER SELECTION (MSUS) ALGORITHM**

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</tr>
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<tbody>
<tr>
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<td>( S = \emptyset ) (emptyset)</td>
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| Step 3 | Algorithm termination: If \( |S| < N_t \), then go to step 2. |
Fig. 2. Average Sum-Rate versus SNR, where $N_t = 4$, $N_r = 2$, $K = 4$ and $B = 4$.

Fig. 3. Average Sum-rate versus Number of users, where $N_t = 4$, $N_r = 2$, $B = 4$ and $\text{SNR} = 10 \text{dB}$.

compared with PU2RC and ZFBF-QBC\(^{2}\). The proposed unitary preprocessing schemes outperform the PU2RC and ZFBF-QBC especially in small and medium user range. MGUS algorithm outperforms MSUS algorithm because the MGUS algorithm utilizes both $p_k$ and $\eta_k$ in selection process at the cost of increased computational complexity. PU2RC, however, achieves the highest sum-rate performance with large user pool ($K \geq 20$) since it compensates the performance loss from preprocessing gain with large mutliuser diversity gain.

V. CONCLUSIONS

In this paper, we propose a new low-complexity unitary preprocessing scheme, which includes preferred-beam feedback method, SVD-based unitary preprocessing method and two simple user selection algorithms. The main features of our proposed unitary preprocessing scheme are that user channel and inter-user interference information are effectively conveyed even in low rate feedback channel and unitary preprocessing matrix is computed with considerably reduced computational complexity compared to the previous works. Numerical results have verified that the proposed scheme achieves higher sum-rate performance than the previous linear preprocessing scheme such as ZFBF-QBC, PU2RC and enhanced unitary beamforming scheme, especially at low SNR or with small number of users. Based on the aforementioned features, the proposed low-complexity unitary preprocessing scheme is highly appropriate for practical MU-MIMO systems.

REFERENCES


\(^{2}\)User selection method for ZFBF-QBC [8] has not been explicitly discussed yet. Hence, we apply the conventional greedy user selection algorithm in [13]