Generalized MMSE Beamforming for Downlink MIMO Systems

Hyungjoo Lee*, Illsoo Sohn†, Donghyun Kim*, and Kwang Bok Lee*
*School of Electrical Engineering and INMC, Seoul National University (SNU), Seoul, Korea.
†Wireless Networking and Communications Groups, University of Texas at Austin, Austin, TX, USA.

Abstract—In this paper, a generalized MMSE beamforming is proposed for downlink multiple input multiple output (MIMO) systems. Unlike the conventional MMSE beamforming, cellular environments are considered where each user is randomly distributed in a cell and has the different temporal correlation of fading channel. We apply the proposed generalized MMSE beamforming to two important downlink MIMO scenarios, which are multiuser MIMO (MU-MIMO) and multicell MIMO (MC-MIMO). We derive the closed-form expressions of the generalized MMSE beamforming for both scenarios using convex optimization technique. The derived beamforming solution captures the impacts of the random geometry and different temporal correlations of users. Numerical results verify that the generalized MMSE beamforming achieves lower BER and higher average sum-rate than the previous beamforming schemes developed for MU-MIMO and MC-MIMO including the conventional MMSE beamforming.

Index Terms—Multiuser MIMO, Multicell MIMO, linear beamforming, MMSE beamforming

I. INTRODUCTION

With increasing demand of various multimedia services, the next generation wireless communication systems are expected to support much higher data rate than before. Multiuser multiple-input multiple-output (MU-MIMO) has been extensively studied as one of the key spectrally-efficient technologies. Multiple antennas at the base station (BS) enable simultaneous transmissions to multiple users to increase cell capacity. Due to the simplicity of implementations and near-optimal performance, linear beamforming techniques such as zero-forcing beamforming [1], random orthogonal beamforming [2], and minimum mean-square error (MMSE) beamforming [3]–[5] are developed for MU-MIMO systems. On the other hand, multicell MIMO (MC-MIMO) has also been recognized as another key technology. In MC-MIMO, multiple BSs exploit multiple transmit antennas to mitigate the other cell interference through share of user channel information. It has been theoretically proven that multiple BSs achieve higher network rate by joint beamforming [14], [15]. Considering the limited backhaul capacity for information sharing between cells, decentralized MC-MIMO techniques such as direct-channel singular value decomposition (D-SVD), projected-channel SVD (P-SVD) [10], and signal to generating interference plus noise ratio (SGINR) beamforming [11] have been proposed recently.

In particular, MMSE beamforming computes the best beamforming vectors by minimizing the error between the transmitted signal and received signal caused from both interuser interference and noise. MMSE beamforming has an advantage over other beamforming techniques since it provides bit-error rate (BER) for its performance metric while other beamforming techniques provides sum-rate computed from an idealized capacity equation [3]–[5]. This makes the MMSE beamforming easy to implement in practical systems. It has also been shown that the MMSE beamforming achieves better system performance than other beamforming techniques in terms of both BER and sum-rate. However, there are still remaining issues to implement the MMSE beamforming in the emerging cellular systems. Firstly, cellular users are randomly located within a cell and therefore experience different path loss [6], [7]. Therefore, the assumption of the conventional MMSE beamforming that all users have the same average signal-to-noise-ratio (SNR) is clearly invalid in this case. Random geometry of the cellular users, i.e., spatial heterogeneity, causes different average SNRs of users. Secondly, a block fading channel model where the channel remains static in each time slot and independently changes over slots is assumed in the conventional MMSE beamforming. However, wireless channels vary during each time slot depending on the mobility of users, which degrades the performance.

In this paper, we propose a generalized MMSE beamforming which mitigates the practical issues of the conventional MMSE beamforming. Our system model includes the different average SNR of users due to the random geometry in cellular systems. Moreover, the channel variation during each time slots are considered in computing beamforming vectors by introducing the temporal correlation of fading channels. Hence, the generalized MMSE beamforming technique can be easily applied to the diverse cellular systems where cellular users are randomly located and the temporal correlation of fading channel at each user exists.

II. GENERALIZED MMSE BEAMFORMING FOR MU-MIMO

In this section, we consider multiuser-MIMO (MU-MIMO) systems. Subsection II-A describes our cellular system model. In subsection II-B, we formulates optimization problem and derive mathematical solution. And then, we propose a generalized MMSE beamforming for MU-MIMO. Finally, we give simulation results in subsection II-C.
A. System Model

Fig. 1 illustrates MU-MIMO system model. We consider a single cell MIMO downlink channel where the base station (BS) has $M$ transmit antennas and each of $K$ users has a single receive antenna. Equal power allocation over $M$ selected users is assumed. The signal received by the $k$-th user is represented as

$$ y_k = \sqrt{p_k} (h_k^H \mathbf{w}_k) x_k + \sqrt{p_k} \sum_{j \neq k} (h_k^H \mathbf{w}_j) x_j + n_k $$  \hspace{1cm} (1)

where $x_k$ is the data symbol transmitted through $M$ transmit antennas satisfying $E[|x_k|^2] = 1$, and $h_k$ is the time-varying channel vector ($h_k \in \mathbb{C}^{M \times 1}$). $x_k$ and the elements of $h_k$ are independent and identical decentralized (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance. The scalar random variables with zero mean and unit variance. The scalar value $\varepsilon_k$ ($0 \leq \varepsilon_k \leq 1$) is the temporal correlation coefficient between the elements of the current channel $h_k$ and those of the previous channel $h_k^{(\tau)}$, and can be characterized by $\varepsilon_k = J_0 (2\pi \cdot f_{D,k} \cdot \tau)$, where $J_0 (x)$ is the zeroth-order Bessel function and $f_{D,k}$ is the maximum Doppler frequency of the $k$-th user in Hertz [9], [12]. As shown in the literature [12], the maximum Doppler frequency is dependent on both carrier frequency and user mobility. In particular, user mobility is a determining factor in the maximum Doppler frequency and is different from each other. As a result, each user has the different time correlation coefficient. Therefore, for the MU-MIMO cellular system model, the different correlation coefficients should be also considered as well as user spatial heterogeneity.

B. Problem Formulation and Generalized MMSE Beamforming

In this subsection, we assume that the base station has imperfect channel feedback information of each user: individual average SNR $\rho_k$, the time correlation $\varepsilon_k$, and the previous channel $h_k^{(\tau)}$, but no information of $h_k$. With the imperfect feedback information, the conditional mean square error (MSE) of the $k$-th user, $\text{MSE}_k$, can be defined as

$$ \text{MSE}_k = E_{x_k, \mathbf{w}_k} \left( \frac{1}{p_k} |y_k - \sqrt{p_k} x_k|^2 \right) $$  \hspace{1cm} (3)

We substitute equation (1) into equation (3), and compute the conditional expectations of $y_k$ and $\mathbf{w}_k$ as (4). In equation (a), we separate the noise term. And, equation (b) follows that $\mathbf{w}_k^H \mathbf{w}_k = 1$. By replacing $h_k$ with equation (2), the expectation on the left hand side of (b) is transformed as (c). Equation (d) follows that $x_k$ and the elements of $h_k$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance. Using the assumptions that $E[|x_k|^2] = E[|n_k|^2] = 1$, we finally get equation (4).

Based on equation (4), the beamforming vectors need to be determined to minimize the total MSEs under the constraint of BS power. Therefore, the MMSE based optimization criterion we consider can be formulated as the following problem:

$$ (\mathbf{w}_1^*, \ldots, \mathbf{w}_K^*) = \arg \min_{\mathbf{w}_1, \ldots, \mathbf{w}_K} \sum_{k=1}^{K} \text{MSE}_k \hspace{1cm} \text{s.t.} \| \mathbf{w}_1 \|^2 = \cdots = \| \mathbf{w}_K \|^2 = 1 $$  \hspace{1cm} (5)

In order to find the optimal beamforming vectors, $(\mathbf{w}_1^*, \ldots, \mathbf{w}_K^*)$, the Lagrange dual objective function could be constructed as

$$ L (\mathbf{w}_1, \ldots, \mathbf{w}_K, \lambda_1, \ldots, \lambda_K) = \sum_{k=1}^{K} \lambda_k (\mathbf{w}_k^H \mathbf{w}_k - 1) + \sum_{k=1}^{K} \text{MSE}_k $$  \hspace{1cm} (6)

where $\lambda_1, \ldots, \lambda_K$ are the Lagrange multipliers. Karush-Kuhn-Tucker (KKT) conditions for the Lagrange dual problem are

1Note that in (3), we modify the conventional MSE definition, $\text{MSE}_k = E[|y_k - x_k|^2]$, due to the difference with the conventional MU-MIMO system model. The conventional system model considers the linear beamforming vector, $\mathbf{w}_k$, that includes transmit power factor, that is, $\| \mathbf{w}_k \|^2 = 2$. However, for simplicity, our system model utilizes the received average SNR, $\rho_k$, that contains the effects of both transmit power and path loss. Thus, using the conventional MSE definition without modification leads to inappropriate results. For example, in the low SNR regime ($\rho_k << 1$), the conventional MSEs of all users are approximately equal to one, $E[|y_k - x_k|^2] \approx 1$, because $y_k$ is almost zero. Therefore, we consider the modified MSE definition to reflect the impact of the difference.

Fig. 1. MU-MIMO system model.
given as following equations (7)-(10).
\[ w_i^*H w_i^* = 1 \quad \cdots \cdots \text{Primal constraint,} \]  
\[ \lambda_i^* \geq 0 \quad \cdots \cdots \text{Dual constraint,} \]  
\[ \lambda_i^* \left[ w_i^{*H} w_i^* \right] = 0 \quad \cdots \cdots \text{Complementary slackness,} \]  
\[ \frac{\partial}{\partial w_i^*} L (w_i^*, \cdots, w_K^*, \lambda_1^*, \cdots, \lambda_K^*) = 0 \]  
\[ \cdots \cdots \text{Gradient of Lagrangian vanishes.} \]  
Since the problem is convex and slater’s condition is satisfied, KKT conditions provide the optimal solution for (5). We take derivative of (6) with respect to \( w_1, \ldots, w_K \) and equate them to zero. Therefore, the optimal beamforming vectors, \( w_1^*, \ldots, w_K^* \), could be solved as \(^2\)
\[ w_i^* = \varepsilon_i \left( \lambda_i + \frac{1}{\rho_i} I \right) + \sum_{k=1}^{K} \varepsilon_k \cdot h_k^T \cdot h_k + (1 - \varepsilon_i^2) \cdot I \right]^{-1} h_i^T. \]  
Note that from (11), we can see the effects of user spatial heterogeneity, \( \rho_i \), and different correlation coefficient, \( \varepsilon_i \), as a part of the beamforming design problem. When the correlation coefficients of all users are equal to one, \( \varepsilon_i = 1 \) (block fading channel model), or when the average SNRs of all user are same, similar expressions derived in [3][5] can be obtained as \( w_i^* = \left( K \bar{\sigma}_e + \frac{K}{\rho} I + \sum_{k=1}^{K} h_k^T \cdot h_k \right)^{-1} h_i^T \), where \( \bar{\rho}, \bar{\sigma}_e \) are the same average SNR and channel error of users. However, in the conventional MMSE beamforming [3][5], the optimal beamforming vector has not explicitly been discussed when users have different SNRs and temporal correlations. Hence, in the numerical results of MU-MIMO, as a simple approach, we use the mean values of users’ different SNRs and temporal correlations for the conventional MMSE beamforming instead of \( \bar{\rho} \) and \( \bar{\sigma}_e \).

In addition, we can find that the optimal beamforming vector, \( w_i^* \), consists of four elements: Lagrange multiplier related with power constraint, SNR of \( i \)-th user, the desired channel of \( i \)-th user, and the channel information of other users. These terms except for user’s SNR are directly affected by the different temporal correlation coefficients of each user, \( \varepsilon_i \). It means that the optimal beamforming vector is constructed based on the accuracy of user channel information because the time correlation coefficient indicates the accuracy of channel information in a different point of view. And, the effect of user SNR, \( \rho_i \), goes to zero at the high SNR regime. This behavior is consistent with that of general MMSE beamforming.

To calculate the Lagrange multipliers, we define Hermit matrix \( A \) as
\[ A = \sum_{k=1}^{K} \left[ \varepsilon_k^2 \cdot h_k^T \cdot h_k + (1 - \varepsilon_k^2) \cdot I \right]. \]  
Then, the matrix \( A \) is decomposed by singular value decomposition (SVD) as \( U \Lambda U^H \). According to power constraint from (5), the value of \( \lambda_i \) can be found as (13). In equation (13), \( \mu_j \) are the singular values of matrix \( A \) and \( u_j \) are the column vectors of matrix \( U \). We note that \( \lambda_i \) from the KKT conditions is either the positive value satisfying the power constraint with equality (\( w_i^{*H} w_i^* = 1 \)) or zero. Therefore, all the beamforming vectors with Lagrange multipliers could be solved out.

C. Numerical Results in MU-MIMO

In this subsection, we present numerical results to evaluate performance of the generalized MMSE beamforming in terms of BER and the average sum-rate. Based on 3GPP-LTE standard, the design specifications are considered in Table 1.\(^3\) Simulation results are averaged over 1,000 independent

\(^2\)In this case, we treat \( w \) as an independent vector similar to [13]. As a result, given an arbitrary matrix \( X \) and the vector \( w \), we can apply the equation, \( \frac{\partial}{\partial w} (X \cdot w) = \frac{\partial}{\partial w} (w \cdot X) = X. \)

\(^3\)We set that the control delay is 5ms and the mobility of user is up to 30km/h. Therefore, the maximum doppler shift is 55.5 Hz and the normalized maximum Doppler shift is obtained by multiplying the maximum Doppler shift and the delay time.
\[ w^*_k^H w^*_i = \varepsilon_k^2 \cdot h_k^{(r)H} \left[ \left( \lambda_i + \frac{1}{\rho_i} \right) \cdot I + A \right]^{-1} \cdot \left[ \left( \lambda_i + \frac{1}{\rho_i} \right) \cdot I + A \right]^{-1} h_k^{(r)} \]

\[ = \varepsilon_k^2 \cdot h_k^{(r)H} \cdot U \left[ \left( \lambda_i + \frac{1}{\rho_i} \right) \cdot I + \Lambda \right]^{-1} \left[ \left( \lambda_i + \frac{1}{\rho_i} \right) \cdot I + \Lambda \right]^{-1} U^H \cdot h_k^{(r)} \]

\[ = \varepsilon_k^2 \sum_{j=1}^{M} \frac{|h_k^{(r)H} u_j|^2}{\left( \lambda_i + \frac{1}{\rho_i} + \mu_j \right)^2}, \quad (13) \]

### TABLE I

<table>
<thead>
<tr>
<th>Carrier Frequency</th>
<th>2 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna Array</td>
<td>(M = 2) or (M = 4)</td>
</tr>
<tr>
<td>Bandwidth (BW)</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Subband BW</td>
<td>1.08 MHz</td>
</tr>
<tr>
<td>Frame Duration</td>
<td>10 msec</td>
</tr>
<tr>
<td>Subframe length</td>
<td>1 msec</td>
</tr>
<tr>
<td>CSI Feedback</td>
<td>2 msec</td>
</tr>
<tr>
<td>Control Delay</td>
<td>4-6 msec</td>
</tr>
<tr>
<td>Mobility</td>
<td>30 km/h</td>
</tr>
</tbody>
</table>

channel realizations and user distributions.

In Fig 2-3, we consider MU-MIMO systems with \(M = 2\) and \(M = 4\). We assume that users are randomly distributed in a cell and user mobility is also randomly generated for each user. For our simulation, cell radius of 1,000m, path loss exponent of 4, and noise power of -100dBm are set.

In Fig. 2, we show how BER performance of different MU-MIMO systems varies with BS transmit power. For simulation, we assume a QPSK modulation and compute the average uncoded BER. The result indicates that the improvement in BER performance is achieved using the generalized MMSE beamforming technique when compared with the conventional MMSE beamforming with mean value and ZFBF. That is, the generalized MMSE beamforming technique outperforms all the other MU-MIMO techniques irrespective of both BS transmit power and the number of transmit antennas.

Fig. 3 compares sum rate performances of different MU-MIMO systems. As we can see from the figure, the generalized MMSE beamforming outperforms ZFBF and the conventional MMSE beamforming in all regions of BS transmit power. The conventional MMSE beamforming with mean value shows the low average sum-rate performance. It means that improper optimization value for MMSE beamforming vector could reduce system performance. And, ZFBF becomes inefficient in the region of low BS transmit power by its nature. Thus, ZFBF shows low sum-rate performance than the generalized MMSE beamforming.

### III. GENERALIZED MMSE BEAMFORMING FOR MC-MIMO

In this section, we consider multicell-MIMO (MC-MIMO) systems. First, we describe MC-MIMO system model focusing on differences with MU-MIMO system model. And then, the generalized MMSE beamforming derived in Section (II-B) is also applied to MC-MIMO. After that, we explain the differences with MU-MIMO and the effects of interference power form adjacent cells as well as SNR and the temporal correlation. Finally, through simulation results, we confirm that the generalized MMSE beamforming technique also offers performance improvement in terms of both sum-rate and BER.
A. System Model

We consider a downlink MC-MIMO system comprised of \( L \) cells. The BS and each user are equipped with \( M \) transmit antennas and a single receive antenna, respectively. The \( k \)-th transmitter (the BS in the \( k \)-th cell) communicates with the \( k \)-th receiver (the MS in the \( k \)-th cell) using the beamforming vector \( \mathbf{w}_k \). For simplicity, a single user is already selected by a user scheduler at the given time and frequency in each cell.

The received signal \( y_k \) at the \( k \)-th receiver is expressed as

\[
y_k = \sqrt{\rho_k} \left( \mathbf{h}_{k,k}^H \mathbf{w}_k \right) x_k + \sum_{j \neq k} \sqrt{\eta_{k,j}} \left( \mathbf{h}_{k,j}^H \mathbf{w}_j \right) x_j + n_k,
\]

where \( \mathbf{h}_{k,j} \) denotes the time-varying channel vector between the \( j \)-th transmitter and the \( k \)-th receiver, \( x_k \) is the data symbol transmitted through \( M \) transmit antennas of the \( k \)-th transmitter and \( n_k \) is the additive complex Gaussian noise with unit variance. The elements of \( \mathbf{h}_{k,j} \) and \( x_k \) are assumed to be independent and identically decentralized (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance. \( \rho_k \) is the SNR of the \( k \)-th cell, and \( \eta_{k,j} \) denotes the interference to noise ratio (INR) for the interference that the \( j \)-th transmitter causes to the \( k \)-th receiver.

Note that in (14), we apply the same assumptions as Section II-A. Therefore, \( \rho_k \) and \( \eta_{k,j} \) are the different SNR of the \( k \)-th cell and the different INR from the \( j \)-th transmitter to the \( k \)-th receiver, respectively, reflecting the spatial heterogeneity of the \( k \)-th user. And, the channel vector \( \mathbf{h}_{k,j} \) is also temporally correlated and can be identically given as (2).

B. Problem Formulation and Generalized MMSE Beamforming

As described in Section II-B, we explain a similar derivation to formulate objective problem and propose the generalized MMSE beamforming for MC-MIMO. We assume that the multiple BSs can share user channel information to alleviate interference from other adjacent cells. Therefore, each BS has user channel feedback information of other cell as well as its own cell: individual average SNR, \( \rho_k \), all average INRs, \( \eta_{k,j} \), the time correlation, \( \varepsilon_k \), and all previous channels, \( \mathbf{h}_{k,j}^{(r)} \), but no information of \( \mathbf{h}_w \). With the shared imperfect feedback information, the conditional MSE \( \mathbf{K}_k \) can be derived as (15).

For derivation of equation (15), the similar processes in (3)-(4) are performed. In the processes, we calculate the conditional expectations of \( x_k \) and \( \mathbf{h}_w \). Using the assumption that \( x_k \) and the elements of \( \mathbf{h}_w \) are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unit variance, we obtain the final expression of user’s MSE. From (15), the objective function of the MMSE optimization for MC-MIMO can be formulated as

\[
(w_1^*, \ldots, w_K^*) = \arg \min_{w_1, \ldots, w_K} \frac{1}{K} \sum_{k=1}^{K} \text{MSE}_k
\]

\[
s.t. \|w_1^*\|^2 = \cdots = \|w_K^*\|^2 = 1.
\]

To obtain the optimal beamforming vectors for (16), we use the Lagrange dual objective function with KKT conditions as (6) and take derivative with respect to \( w_1, \ldots, w_K \). As a result, the final optimal beamforming vectors, \( w_1^*, \ldots, w_K^* \), are computed as

\[
w_i = \varepsilon_i \left( A_i + \frac{1}{\rho_i} \right) I + \sum_{k=1}^{K} \frac{\eta_{i,k}}{\rho_i} \left( \varepsilon_i^2 \mathbf{h}_{i,k}^{(r)} \mathbf{h}_{i,k}^{(r)}H + (1 - \varepsilon_i^2) I \right) \mathbf{h}_{i,k}^{(r)}.
\]

The values of the Lagrange multipliers can be gained through similar calculations in (12)-(13). Therefore, the generalized MMSE beamforming derived in Section II could be applied to MC-MIMO system.

C. Numerical Results in the Decentralized MC-MIMO

In Fig. 5, we consider the two cell (\( L = K = 2 \)) and three cell (\( L = K = 3 \)) MC-MIMO systems with \( M = 2 \). We assume that users are randomly located between \( d = 700 \) and \( d = 1000 \) for a representation of a cell edge user. And, as the assumptions of MU-MIMO systems, cellular users have different SNRs, INRs, and time correlations.

In Fig. 4, we show BER performance of the generalized MMSE beamforming compared with D-SVD, P-SVD, and SGINR beamforming. As expected, the generalized MMSE beamforming attains the low average BER and performance difference becomes larger especially at the region of high BS transmit power. This improvement demonstrates that the generalized MMSE beamforming technique provides more benefits to users near the cell boundary since it can efficiently control intercell interference to alleviate performance degradation.

IV. CONCLUSION

In this paper, we have developed a generalized MMSE beamforming technique for MU-MIMO systems. And we have applied the generalized MMSE beamforming technique to
MC-MIMO systems. In both cases, we have considered the realistic cellular systems with the user’s random geometry and temporal correlation. Surprisingly, we have shown that the great improvement in the average sum rate and BER performance is achieved using the generalized MMSE beamforming technique.

REFERENCES


