Statistical Transmit Antenna Subset Selection for Limited Feedback MIMO Systems

Chang Soon Park and Kwang Bok Lee

School of Electrical Engineering and Computer Science, Seoul National University
Shinlim-dong, Kwanak-gu, Seoul, 151-742, Korea (e-mail: parkcs@mobile.snu.ac.kr; klee@snu.ac.kr)

Abstract—In this paper, we investigate statistical transmit antenna subset selection to improve the capacity of wireless multiple-input multiple-output (MIMO) systems with linear receivers. In order to reduce feedback overhead, we allocate equal power and equal rate to all selected transmit antennas. The equal data rate is determined by the minimum post-detection signal-to-noise ratio (SNR) and adjusted every channel instance. To reduce feedback overhead further and channel estimation burden for all transmit antennas every channel instance, the antenna selection is based on the long-term statistics of spatially correlated MIMO fading channels. In particular, we first derive an analytical closed-form expression for the expectation of the lower bound on the capacity using the smallest eigenvalue distribution of a Wishart matrix. Then, we propose a transmit antenna subset selection criterion of maximizing this average lower-bound capacity. Numerical results show that the proposed selection scheme improves the performance with small feedback overhead.

I. INTRODUCTION

Recent information-theoretic results have shown that multiple-input multiple-output (MIMO) systems, which employ multiple antennas at both the transmitter and receiver, provide considerable capacity enhancement of wireless communication channels [1], [2]. In practical systems, however, the hardware cost of RF chains for multiple antennas may limit the number of antennas. In order to overcome this limitation while achieving satisfactory MIMO gain, various (receive or transmit) antenna selection schemes have been investigated in [3]–[8], where the antenna subset is adapted to the instantaneous channel conditions. While the number of selected antennas is assumed to be fixed in [3]–[7], that of the transmitter is varied in [8], and the increased degrees of freedom in the number of transmitted substreams have been shown to improve the performance. However, the transmit antenna selection based on the instantaneous channel conditions may cause the performance degradation due to the limited feedback channel bandwidth, limiting its application to practical systems over fast fading channels. Furthermore, it requires the channel estimation process for all transmit antennas at every transmission time interval. In [9]–[11], the transmit antenna selection is based not on the instantaneous channel conditions but on the long-term channel statistics. However, the feedback overhead for different rates for all selected transmit antennas still remains and the number of selected antennas is not variable in [9]. The works in [10] and [11], respectively, use the selection criteria of maximizing the minimum signal-to-noise ratio (SNR) margin and of minimizing the average probability of error, and those still leave the need to investigate the antenna selection for capacity improvement.

In this paper, we investigate transmit antenna subset selection to improve the capacity (not error-rate performance) of spatially correlated MIMO fading channels with small feedback overhead. We consider the linear receiver with low complexity which uses the zero-forcing (ZF) or minimum mean square error (MMSE) detection scheme. In order to reduce feedback overhead and system complexity, we allocate equal power and equal rate to all selected transmit antennas, from which independent data substreams are transmitted. The equal rate is determined by the minimum post-detection SNR, and adjusted every channel instance. To reduce feedback overhead further and channel estimation burden for all transmit antennas at every instance, we select a transmit antenna subset among all possible subsets based on the long-term statistical properties of MIMO channels. In particular, we first derive an analytical closed-form expression for the expectation of the lower bound on the capacity using the smallest eigenvalue distribution of a Wishart matrix. Then, we propose a transmit antenna subset selection criterion of maximizing the average lower-bound capacity. The performance of the proposed scheme is evaluated, and compared with that of the non-adaptive transmission from a fixed transmit antenna subset.

II. SYSTEM AND CHANNEL MODELS

A point-to-point MIMO communication system with \(N_t\) transmit antennas and \(N_r\) receive antennas is depicted in Fig. 1. When the number of selected transmit antennas is denoted by \(M\), \(M\) can be taken from 1 to \(\min(N_t, N_r)\). The subset, which consists of indices for selected transmit antennas, is denoted by \(S\). When all available transmit antennas are selected, for example, \(M = N_t\) and \(S = \{1, 2, \ldots, N_t\}\). At the receiver, all \(N_r\) receive antennas are assumed to be used to detect the transmitted data signals. Throughout this paper, the superscripts \([\cdot]^T\) and \([\cdot]^H\) denote the transpose and conjugate transpose, respectively. Moreover, \([A]_{i,j}\) denotes the element in the \(i\)th row and \(j\)th column of a matrix \(A\), and \(E[\cdot]\) represents the expectation operator.

The \(N_t \times N_r\) channel matrix is modeled as [12]

\[
H = R_{Rr}^{1/2}H_{tx}^{}R_{Rr}^{1/2}
\]

(1)

where \(R_{Rr}\) and \(R_{tx}\) are the \(N_r \times N_r\) receive and \(N_t \times N_t\) transmit correlation matrices, respectively. The elements of \(H_{tx}\) in (1) are assumed to be independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance. The \(N_t \times M\) channel matrix for transmit antenna subset \(S\) may be modeled as

\[
\tilde{H}(S) = R_{Rx}^{1/2}H_{tx}^{}(S)R_{Rx}^{1/2}(S)
\]

(2)

where \(\tilde{H}_{tx}(S)\) is an \(M \times M\) principal submatrix of \(R_{tx}\), and \(\tilde{H}_{tx}(S)\) is an \(N_t \times M\) matrix whose elements have the same distributions as those of \(H_{tx}\).

Correspondingly, the received signal vector for a given \(S\), may be expressed as

\[
y = \tilde{H}(S)x + n
\]

(3)

where \(x\) is an \(M \times 1\) transmit symbol vector. The covariance matrix of \(x\) is given by \(E[x x^H] = (E/M)I_M\), where \(E\) is the total average transmit energy and \(I_M\) is the \(M \times M\) identity matrix. This implies that independent data symbols are transmitted from all selected transmit antennas. The vector \(n\) denotes an \(N_r \times 1\)
circularly symmetric complex Gaussian noise vector with zero mean and covariance matrix $\mathbb{E}[\mathbf{m}\mathbf{m}^\dagger] = \sigma^2 \mathbf{I}_d$. 

We assume that the receiver has perfect knowledge of the instantaneous and statistical channel state information (CSI). Based on the statistical CSI, such as fading correlation matrices and average SNR, the transmit antenna subset is selected at the receiver and its information is fed back to the transmitter, as depicted in Fig. 1. This feedback information is not necessary to adapt to the varying channel conditions, if long-term statistics of CSI is not changed. Only a single information for equal rate adaptation based on the instantaneous CSI is fed back to the transmitter every channel instance.

III. STATISTICAL TRANSMIT ANTENNA SUBSET SELECTION

A. Capacity for Equal Rate Allocation

First, we briefly describe the detection process and the corresponding post-detection SNR (or SINR), when linear receivers are used. We assume that $\mathbf{y}$ in (3) is multiplied by the corresponding post-detection SNR for the 4th data symbol ($k = 1, 2, \ldots, M$) is calculated as [4]

$$
\operatorname{SNR}^p_{k}(S) = \frac{E_s}{M \sigma^2 \left[ (\mathbf{H}(S)\mathbf{y}(S) + I_{M_0})^{-1} \right]_{k,k}.}
$$

(4)

For the MMSE receiver, $\mathbf{T} = (\mathbf{H}(S)^\dagger \mathbf{H}(S) + (M \sigma^2 / E_s) I_{M_0})^{-1} \mathbf{H}(S)^\dagger$ and the corresponding post-detection SINR for the 4th data symbol is calculated as [10]

$$
\operatorname{SNR}_{\text{MMSE}}(S) = \frac{1}{M \sigma^2 \left[ (\mathbf{H}(S)^\dagger \mathbf{H}(S) + I_{M_0})^{-1} \right]_{k,k}.}
$$

(5)

The allocation of different rates over different subchannels for selected transmit antennas may increase feedback overhead, and need different coding rates and modulation levels over these subchannels, resulting in high complexity. In order to reduce feedback overhead and system complexity, we allocate equal rate across all selected transmit antennas. This equal rate should be the capacity value computed using the minimum SNR, since the data signal can be sent reliably with an arbitrarily small probability of error when the rate is smaller than the capacity value [13]. Thus, the instantaneous overall capacity for equal rate allocation for a given $S$ is calculated as

$$
C_{\text{ER}}(S) = M_s \log_2 (1 + \operatorname{SNR}_{\text{min}}(S))
$$

(6)

where $\operatorname{SNR}_{\text{min}}$ is the minimum SNR among the values of (4) for the ZF receiver, and (5) of the MMSE receiver.

Here, it should be noted that, in the original sense, the “channel capacity” means maximum mutual information over all distributions on the input signal with the optimal power allocation [13]. The overall capacity in this paper, however, means the sum of rates, each one of which is the maximum achievable rate for each data subset when data signals are transmitted with equal power and equal rate, and linear detection is used.

B. Average Lower-Bound Capacity Based Selection

In this subsection, we propose the criterion for selecting transmit antenna subset based on the long-term statistics of MIMO fading channels. The distribution of $\operatorname{SNR}_{\text{min}}(S)$ in (6) should be derived to find the analytical expectation of $C_{\text{ER}}(S)$ with respect to random fading channels. However, the exact solution of that problem is difficult to find. Thus, we use a lower bound on $\operatorname{SNR}_{\text{min}}(S)$ (for both ZF and MMSE receivers), which is given as [10]

$$
\operatorname{SNR}_{\text{min}}(S) \geq \frac{E_s}{M \sigma^2} \lambda_{\text{min}}(\mathbf{R}_{\text{xx}}(S)) \lambda_{\text{min}}(\mathbf{R}_{\text{xx}}(S))
$$

(7)

where $\lambda_{\text{min}}(\mathbf{A})$ denotes the smallest (real) eigenvalue of a Hermitian matrix $\mathbf{A}$. Correspondingly, the $C_{\text{ER}}(S)$ in (6) is lower bounded as

$$
C_{\text{ER}}(S) \geq C_{\text{ER,LB}}(S)
$$

(8)

with the lower bound $C_{\text{ER,LB}}(S)$ given as

$$
C_{\text{ER,LB}}(S) = M_s \log_2 \left( 1 + \frac{E_s}{M \sigma^2} \lambda_{\text{min}}(\mathbf{R}_{\text{xx}}(S)) \lambda_{\text{min}}(\mathbf{R}_{\text{xx}}(S)) \mathbf{H}(S)^\dagger \mathbf{H}(S) \right)
$$

(9)

Then, the expectation of $C_{\text{ER,LB}}(S)$ with respect to random fading channels is expressed as

$$
E[C_{\text{ER,LB}}(S)] = M_s \int_0^\infty \log_2 (1 + \rho(S) \lambda) f_\lambda^{\text{MMSE}}(\lambda) d\lambda
$$

(10)

where $\rho(S) = (E_s / (M \sigma^2))^2 \lambda_{\text{min}}(\mathbf{R}_{\text{xx}}(S)) \lambda_{\text{min}}(\mathbf{R}_{\text{xx}}(S))$ and $f_\lambda^{\text{MMSE}}(\lambda)$ denotes the probability density function (pdf) of $\Lambda_{\text{mm}} = \lambda_{\text{min}}(\mathbf{H}(S)^\dagger \mathbf{H}(S))$.

Note that $\Lambda_{\text{mm}}$ is its smallest eigenvalue. Using the results in [14] and considering that the elements of $\mathbf{H}(S)$ have unit variance, the joint pdf of the ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M$ of $\mathbf{H}(S)^\dagger \mathbf{H}(S)$ is expressed as

$$
f_{\lambda_{\lambda_1,\lambda_2,\ldots,\lambda_M}}(\lambda_1, \lambda_2, \ldots, \lambda_M) = K_{\lambda_{M,N}} \prod_{i=1}^{M-N} \Gamma(M-i+1) \Gamma(M_i)
$$

(11)

where

$$
K_{\lambda_{M,N}} = \left( \prod_{i=1}^{M-N} \Gamma(N_i - i + 1) \Gamma(M_i - i + 1) \right)^{-1}
$$

and $\Gamma(\alpha) \triangleq \int_0^\infty e^{-t} t^{\alpha-1} dt$ denotes the gamma function. Using the symmetry and applying the transformation $x_i = \lambda_i - \lambda$, the pdf of the smallest eigenvalue $\Lambda_{\text{mm}}$ can be given as [14]

$$
f_{\lambda_{\lambda_{\text{mm}}}}(\lambda) \triangleq c_{\lambda_{\lambda_{\text{mm}}}}(\lambda) e^{M-N} \lambda^{N-M} \int_{\lambda}^{\infty} \prod_{i=1}^{M-N} \left( \lambda_i - \lambda \right)^{M_i-1} \lambda^{N_i-1} e^{-\lambda} d\lambda
$$

(12)

where $R = \{x_1, x_2, \ldots, x_M, \lambda\}$ is the integration region. From (12), it can be easily shown that $f_{\lambda_{\lambda_{\text{mm}}}}(\lambda)$ is expressed as

$$
f_{\lambda_{\lambda_{\text{mm}}}}(\lambda) = e^{M-N} \sum_{n=0}^{M-N} \lambda^n c_n^{M-N}
$$

(13)

where $D = M(N_i - M_i)$ is the degree of the polynomial part of $f_{\lambda_{\lambda_{\text{mm}}}}(\lambda)$ and $c_n^{M-N}$ is the constant coefficient of $\lambda^n$ for a given $M$. The closed-form expressions for $f_{\lambda_{\lambda_{\text{mm}}}}(\lambda)$ and $c_n^{M-N}$ in (13) are derived in Appendix A. In this Appendix, the solutions for $M = 1, 2, 3$, and 4 with arbitrary $N_i$ are derived, since $M_i$ seems to be seldom more than four in practical systems. To the best of authors’ knowledge, these simple and explicit expressions, which do not require complicated integration and consist of a single exponential function multiplied by a polynomial of degree $D$, have not been found in the previous works.1 Furthermore, the expression of (13) can lead to the derivation of a closed-form average lower-bound capacity as described in the following.

1 In [15], a similar expression has been derived for the pdf of the largest (not smallest) eigenvalue, which is a finite linear combination of elementary gamma pdf’s and those coefficients are computed by using a software tool for some values of $M$ and $N_i$. 

Using (13), $E[C_{\text{LR},\text{LB}}(S)]$ in (10) is given as

$$E[C_{\text{LR},\text{LB}}(S)] = M \sum_{n=0}^{\infty} c^{(M)}_n \int_0^{\infty} \log_2(1 + \rho(S)\lambda)e^{-\mu(S)d\lambda} d\lambda.$$  

The integral term in (14) is derived in Appendix B. Using this result, $E[C_{\text{LR},\text{LB}}(S)]$ is found as

$$E[C_{\text{LR},\text{LB}}(S)] = \frac{\sum_{\alpha=0}^{M-1} \left(-1\right)^\alpha e^{\alpha x}}{(2n)!} \prod_{i=1}^{n} \frac{1}{(n-k)!} \frac{1}{\mu(S)^k} \sum_{i=1}^{M} \frac{\Gamma(\mu(S))}{i!}$$

where $\mu(S) = M/\rho(S)$ and $\Gamma(\alpha, x) \equiv \int_0^x e^{-t}t^{\alpha-1}dt$ denotes the incomplete gamma function [16].

Now, we propose the selection criterion, which is to select the transmit antenna subset $S^*$ that maximizes the average lower-bound capacity in (15). The straightforward application of this criterion requires at most $\sum_{n=1}^{M} \binom{N_t}{n}$ (i.e., the number of all possible S's) computations of $E[C_{\text{LR},\text{LB}}(S)]$ in (15). From (14), however, it can be seen that more $\rho(S)$ (depending on $\lambda^\alpha(R_n(S))$) leads to more $E[C_{\text{LR},\text{LB}}(S)]$ for a given $M_t$. This implies that a simple comparison of $\rho(S)$'s can provide the best transmit antenna subset for a given $M_t$ without computation of $E[C_{\text{LR},\text{LB}}(S)]$. Thus, we first choose $\min(N_t', N_t)$ transmit antenna subsets, each one of which is the subset corresponding to maximum $\rho(S)$ for each $M_t$, and is denoted by $S^*(M_t)$ ($M_t = 1, 2, \ldots, \min(N_t', N_t)$). Then, we select the one subset $S^*$ among $S^*(M_t)$'s that maximizes (15). This two-step selection procedure is described as follows.  

Step 1) Select $S^*(M_t)$'s for $M_t = 1, 2, \ldots, \min(N_t', N_t)$ as

$$S^*(M_t) = \arg \max_{S \in \mathcal{S}, |S| = \min(N_t', N_t)} \rho(S)$$

where $\mathcal{S}$ denotes the number of elements of a set $S$.

Step 2) Select $S^*$ among $S^*(M_t)$'s obtained from Step 1 as

$$S^* = \arg \max_{M_t = 1, 2, \ldots, \min(N_t', N_t)} E[C_{\text{LR},\text{LB}}(S)]$$

where $c^{(M)}_n$'s are given by (A.11).

As shown in (16), the selection of $S^*(M_t)$ in Step 1 for a given $M_t$ is dependent on the smallest eigenvalue of the corresponding transmit correlation submatrix. On the other hand, the selection of $S^*$ in Step 2, where $M_t$ is also chosen, is dependent on the average SNR ranges, as shown in the following section.

IV. NUMERICAL RESULTS

In this section, the performance of the statistical transmit antenna selection described in Section III is provided. The average SNR is defined to be $E[\frac{1}{2}\sigma^2]$. The average capacity $E[C_{\text{LR}}(S^*)]$ of the proposed selection scheme is calculated by averaging the capacity $C_{\text{LR}}(S^*)$ in (6), where $S^*$ is selected by the criterion in (16) and (17), over 10,000 independently generated channel realizations (i.e., by Monte Carlo simulations). For performance comparisons, we also provide the average capacity values of $C_{\text{LR}}(S)$ (i.e., $E[C_{\text{LR}}(S)]$) obtained by Monte Carlo simulations for the transmissions with fixed transmit antenna subset $S$, which is called the non-adaptive scheme in this section. Furthermore, we assume no fading correlation among receive antennas (i.e., $R_{\text{Rx}} = I_{N_r}$), and the ZF detection is assumed to be used. Prior to the performance comparisons between the proposed and non-adaptive schemes, we first illustrate $P'_{\text{mac}}(x)$ in (13) in Fig. 2(a), and compare $E[C_{\text{LR}}(S)]$ and its lower bound $E[C_{\text{LR},\text{LB}}(S)]$ for fixed transmit antenna subsets S's in Fig. 2(b), when $N_r = N_t = 3$ and $R_{\text{Tx}} = I_{3\times3}$ (i.e., i.i.d. fading channels). Fig. 2(a) shows that smaller $M_t$ provides a higher probability of $\Lambda_{\text{min}}$ being larger values, and thus increases diversity gain. The values of $E[C_{\text{LR},\text{LB}}(S)]$ in Fig. 2(b) are illustrated using the analytical expression (15), and can be shown to be equal to those obtained by Monte Carlo simulations, which are not included in Fig. 2(b). Note that $E[C_{\text{LR},\text{LB}}(S)]$ becomes equal to $E[C_{\text{LR}}(S)]$ in the case of $M_t = 1$, since the lower bound on SNR $\text{min}$(S) in (7) is identical to SNR $\text{min}$(S) for $M_t = 1$. The difference between $E[C_{\text{LR},\text{LB}}(S)]$ and $E[C_{\text{LR}}(S)]$ for $M_t = 2$ is observed to be smaller than that for $M_t = 3$. Fig. 2(b) also shows that the performance for smaller $M_t$ tends to be superior to that for larger $M_t$ at low SNR range, while the reverse trend is observed at high SNR range. This may be explained by using the property of capacity function, which increases logarithmically with SNR increasing. The transmission of fewer data substreams increases each capacity for each data substream resulting from more allocated power to each substream (as seen from $\rho(S)$ in (10)) and increased diversity effects (as shown in Fig. 2(a)). Although the overall capacity is a linear sum of $M_t$ capacity values for $M_t$ data substreams, at low SNR range, increased each capacity described above has a more dominant effect on the overall sum capacity than larger $M_t$. On the other hand, at high SNR range, linear summation of $M_t$ capacity values gives more overall sum capacity, since each capacity for each data substream increases logarithmically with SNR increasing.

Fig. 3 shows the average capacity performance of the proposed selection scheme in the presence of transmit antenna correlation, when $N_r = 6$ and $N_t = 4$. We also provide the performance for the worst cases of fixed S's, each one of which shows the worst performance for each given $M_t$. Using the fading correlation model in [12], the correlation matrix for transmit antennas spaced equally along an axis is given by

$$(R_{\text{Rx}})_{ij} = J_0(\theta(2\pi\lambda_{i\lambda}))/J_0(\lambda_{i\lambda}),$$

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, $\theta$ is the angle spread, $\lambda_{i\lambda}$ is the wavelength, and $d_{\lambda_{i\lambda}}$ denotes the distance between adjacent transmit antennas. It is assumed that $\theta = 3\lambda$ and $d_{\lambda_{i\lambda}} = 3\lambda_{i\lambda}$ in Fig. 3. The proposed scheme is observed to outperform the non-adaptive schemes over all the SNR ranges. The SNR gain of the proposed scheme is about 3.5 dB over the non-adaptive scheme of fixed $S = \{1, 2\}$, 9.4 dB over that of $S = \{1\}$, and 11 dB over that of $S = \{1, 2, 3\}$, at average capacity of 10 bps/Hz.

Fig. 4 compares the average capacity performance of the proposed selection scheme and the non-adaptive schemes for the best cases of fixed S's, each one of which provides the best performance for each given $M_t$. The simulation configurations are the same as those in Fig. 3. As shown in Figs. 3 and 4, the best selection of fixed $S$ significantly outperforms the worst selection of fixed $S$, for each $M_t$ larger than one (note that all fixed $S$'s for $M_t = 1$ provide the same
performance). When we compare the performance in Fig. 4, we first observe that the proposed scheme selects the best $S^t$ ($M_t$)'s for any given value of $M_t$ (by the procedure of Step 1 in (16)), although it does not hold necessarily. Next, although the proposed scheme outperforms the non-adaptive schemes over almost all the SNR ranges, it may not select best $S^t$ among $S^t$ ($M_t$)'s for all $M_t$ (by the procedure of Step 2 in (17)) at some values of average SNR. This is caused by the difference between $C_{\text{ER}}(S)$ in (6) and $C_{\text{ER,RL}}(S)$ in (9), as shown in Fig. 2(b). Fig. 4 also shows that, in the proposed scheme, the statistically selected $M_t$ increases with average SNR increasing, as expected from the results of Fig. 2(b).

V. CONCLUSIONS

We have derived the statistical transmit antenna subset selection scheme to improve the capacity of a limited feedback MIMO system with linear receivers. We have considered the equal power and equal rate data transmission, which reduces feedback overhead. The transmit antenna selection utilizing the long-term statistics of spatially correlated MIMO channels has been developed to reduce the feedback overhead further. We have derived an analytical closed-form expression for the expectation of the lower bound on the capacity, and then proposed the selection criterion of maximizing this average lower-bound capacity. The proposed scheme has been shown to outperform the non-adaptive scheme over almost all the SNR ranges in the presence of transmit antenna correlation.

APPENDIX A

PDF OF THE SMALLEST EIGENVALUE

In this Appendix, the pdf of the smallest eigenvalue of a complex Wishart matrix in (12) is simplified to a closed-form expression of (13), when $N_t \geq M_t$. It is assumed that $N_t \geq M_t$.

For $M_t = 1$, it can be easily shown that $f_{\lambda_{\min}}^{(1)}(\lambda)$ in (12) becomes

$$f_{\lambda_{\min}}^{(1)}(\lambda) = e^{-\lambda} \lambda^{-1} a_k^{(1)}$$

where $a_k^{(1)} = ((N_t - 1)!)^{-1}$. Prior to the derivation of results for $M_t \geq 2$, we define and obtain the following integral as

$$I_{x}(m) = \int_{0}^{\infty} x^m e^{-x} dx = \sum_{k=0}^{m} \frac{m!}{k!} \lambda^k$$

where we use the binomial theorem and the integral result of

$$\int_{0}^{\infty} x^m e^{-x} dx = m!$$

Then, for $M_t = 2$, $f_{\lambda_{\min}}^{(2)}(\lambda)$ in (12) can be calculated as

$$f_{\lambda_{\min}}^{(2)}(\lambda) = K_{2} e^\lambda \lambda^{-3} I_{N_t - 3}(2) = e^{2\lambda} \lambda^{-3} \sum_{k=0}^{N_t - 2} \frac{a_k^{(2)} \lambda^k}{k!}$$

where

$$a_k^{(2)} = \frac{1}{(N_t - 2)!(N_t - k - 2)!} (N_t - k - 1)!$$

Next, for $M_t = 3$, using the symmetry in the integral evaluation, $f_{\lambda_{\min}}^{(3)}(\lambda)$ in (12) can be calculated as

$$f_{\lambda_{\min}}^{(3)}(\lambda) = \frac{e^{3\lambda} \lambda^{-3}}{2(N_t - 1)(N_t - 2)(N_t - 3)!} \left( I_{N_t - 3}(4) I_{N_t - 3}(2) - (I_{N_t - 3}(3))^2 \right)$$

Then, calculating the product and sum of polynomials in (A.5) yields

$$f_{\lambda_{\min}}^{(3)}(\lambda) = e^{3\lambda} \lambda^{-3} \sum_{k=0}^{N_t - 3} \frac{a_k^{(3)} \lambda^k}{k!}$$

where

$$a_k^{(3)} = \frac{1}{2(N_t - 1)(N_t - 2)(N_t - 3)!} \left( \sum_{i=k}^{N_t - 3} \frac{N_t - i - 1}{k} \right)$$

Thus, for $M_t = 1, 2, 3,$ and 4, we obtain

$$c_k^{(M_t)}(\lambda) = a_k^{(M_t)}$$

otherwise.

APPENDIX B

DERIVATION OF INTEGRAL TERM IN (14)

In this Appendix, we derive the integral term in (14). By using the substitution $t = 1 + \rho(S) \lambda$, this integral can be calculated as

$$\int_{0}^{\infty} \log(1 + \rho(S) \lambda) \lambda^x \rho(S)^{\rho(S)m} \lambda^x d\lambda = \frac{e^{\rho(S)}}{(\ln 2) \rho(S)^{\rho(S)m}} \int_{0}^{\infty} (t - 1)^x (\ln t)e^{-\rho(S)\mu dt} = \frac{e^{\rho(S)}}{(\ln 2) \rho(S)^{\rho(S)m}} \int_{0}^{\infty} t^x (\ln t)e^{-\rho(S)\mu dt}$$

where $\mu(S) = M_t \rho(S)$. Next, the integral term of the last result of (B.1) is derived as

$$X_{M_t} = \int_{0}^{\infty} t^x (\ln t)e^{-\rho(S)\mu dt}$$

Therefore, the proposed scheme outperforms the non-adaptive schemes over almost all the SNR ranges in the presence of transmit antenna correlation.

PDF OF THE SMALLER EIGENVALUES

In this Appendix, the pdf of the smallest eigenvalue of a complex Wishart matrix in (12) is simplified to a closed-form expression of (13), when $M_t = 1, 2, 3, $ and 4. It is assumed that $N_t \geq M_t$.

For $M_t = 1$, it can be easily shown that $f_{\lambda_{\min}}^{(1)}(\lambda)$ in (12) becomes

$$f_{\lambda_{\min}}^{(1)}(\lambda) = e^{-\lambda} \lambda^{-1}$$

where $a_k^{(1)} = ((N_t - 1)!)^{-1}$. Prior to the derivation of results for $M_t \geq 2$, we define and obtain the following integral as

$$I_{x}(m) = \int_{0}^{\infty} x^m e^{-x} dx = \sum_{k=0}^{m} \frac{m!}{k!} \lambda^k$$

where we use the binomial theorem and the integral result of

$$\int_{0}^{\infty} x^m e^{-x} dx = m! [16].$$

Then, for $M_t = 2$, $f_{\lambda_{\min}}^{(2)}(\lambda)$ in (12) can be calculated as

$$f_{\lambda_{\min}}^{(2)}(\lambda) = K_{2} e^\lambda \lambda^{-3} I_{N_t - 3}(2) = e^{2\lambda} \lambda^{-3} \sum_{k=0}^{N_t - 2} \frac{a_k^{(2)} \lambda^k}{k!}$$

where

$$a_k^{(2)} = \frac{1}{(N_t - 2)!} (N_t - k - 1)!$$

Next, for $M_t = 3$, using the symmetry in the integral evaluation, $f_{\lambda_{\min}}^{(3)}(\lambda)$ in (12) can be calculated as

$$f_{\lambda_{\min}}^{(3)}(\lambda) = \frac{e^{3\lambda} \lambda^{-3}}{2(N_t - 1)(N_t - 2)(N_t - 3)!} \left( I_{N_t - 3}(4) I_{N_t - 3}(2) - (I_{N_t - 3}(3))^2 \right)$$

Then, calculating the product and sum of polynomials in (A.5) yields

$$f_{\lambda_{\min}}^{(3)}(\lambda) = e^{3\lambda} \lambda^{-3} \sum_{k=0}^{N_t - 3} \frac{a_k^{(3)} \lambda^k}{k!}$$

where

$$a_k^{(3)} = \frac{1}{2(N_t - 1)(N_t - 2)(N_t - 3)!} \left( \sum_{i=k}^{N_t - 3} \frac{N_t - i - 1}{k} \right)$$

Thus, for $M_t = 1, 2, 3,$ and 4, we obtain

$$c_k^{(M_t)}(\lambda) = a_k^{(M_t)}$$

otherwise.

APPENDIX B

DERIVATION OF INTEGRAL TERM IN (14)

In this Appendix, we derive the integral term in (14). By using the substitution $t = 1 + \rho(S) \lambda$, this integral can be calculated as

$$\int_{0}^{\infty} \log(1 + \rho(S) \lambda) \lambda^x \rho(S)^{\rho(S)m} \lambda^x d\lambda = \frac{e^{\rho(S)}}{(\ln 2) \rho(S)^{\rho(S)m}} \int_{0}^{\infty} (t - 1)^x (\ln t)e^{-\rho(S)\mu dt} = \frac{e^{\rho(S)}}{(\ln 2) \rho(S)^{\rho(S)m}} \int_{0}^{\infty} t^x (\ln t)e^{-\rho(S)\mu dt}$$

where $\mu(S) = M_t \rho(S)$. Next, the integral term of the last result of (B.1) is derived as

$$X_{M_t} = \int_{0}^{\infty} t^x (\ln t)e^{-\rho(S)\mu dt}$$

Therefore, the proposed scheme outperforms the non-adaptive schemes over almost all the SNR ranges in the presence of transmit antenna correlation.
where the integration by parts formula is used and $\Gamma(\alpha, x) \int_0^x e^{-t} x^\alpha dt$ denotes the incomplete gamma function [16]. Expanding (B.2) and using $X_i = \mu(S) \Gamma(0, \mu(S))$, it can be shown that

$$X_i = \frac{1}{\mu(S)} \sum_{i=0}^k \binom{k}{i} \frac{1}{\mu(S)^i} \Gamma(k-i, \mu(S)). \quad (B.3)$$

Thus, by substituting (B.3) into (B.1), we get

$$\int_{\min}^\infty \log_2(1 + \rho(S) \lambda) x^{\mu(S)-1} e^{-\lambda x} \, dx = \log(1 + \rho(S) \lambda) - \sum_{i=0}^k \binom{k}{i} \frac{1}{\mu(S)^i} \Gamma(i, \mu(S)). \quad (B.4)$$

REFERENCES


Fig. 1. MIMO system with statistical transmit antenna selection.

Fig. 2. Numerical results for fixed transmit antenna subsets $S$, when $N_t = N_r = 3$ and fading channels are i.i.d. (a) Pdf of $\lambda_{\text{min}}$ (b) Comparison of $E[C_{\text{eq}}(S)]$ and $E[C_{\text{ELEO}}(S)]$.

Fig. 3. Performance comparison with the worst cases of fixed transmit antenna subset $S$ in the presence of transmit antenna correlation, when $N_t = 6$ and $N_r = 4$.

Fig. 4. Performance comparison with the best cases of fixed transmit antenna subset $S$ in the presence of transmit antenna correlation, when $N_t = 6$ and $N_r = 4$. 

Figures